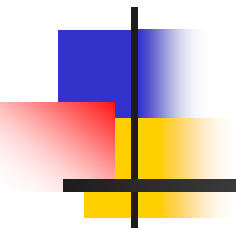
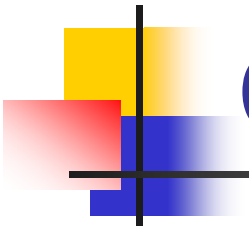


Machine Learning

Lecture 2

Concept Learning



Outline

- Learning from examples
- General-to specific ordering of hypotheses
- Version spaces and candidate elimination algorithm
- Inductive bias

Training Examples for Concept Enjoy Sport

Concept: ”days on which my friend Aldo enjoys his favourite water sports”

Task: predict the value of ”Enjoy Sport” for an arbitrary day based on the values of the other attributes

Sky	Temp	Humid	Wind	Water	Fore-cast	Enjoy Sport
Sunny	Warm	Norr		Warm	Same	Yes
Sunny	Warm	High		Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

instance

Representing Hypothesis

- Hypothesis h is a conjunction of constraints on attributes
- Each constraint can be:
  - A specific value : e.g. *Water=Warm*
  - A don't care value : e.g. *Water=?*
  - No value allowed (null hypothesis): e.g. *Water=Ø*
- Example: hypothesis h

Sky	Temp	Humid	Wind	Water	Forecast
< Sunny	?	?	Strong	?	Same >

Prototypical Concept Learning Task

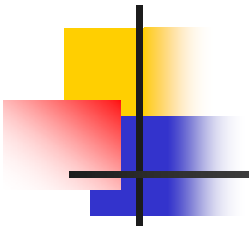
Given:

- Instances X : Possible days decribed by the attributes *Sky, Temp, Humidity, Wind, Water, Forecast*
- Target function c: EnjoySport X $\rightarrow$ {0,1}
- Hypotheses H: conjunction of literals e.g.

  < Sunny ? ? Strong ? Same >
- Training examples D : positive and negative examples of the target function: $<x_1,c(x_1)>,\ldots, <x_n,c(x_n)>$

Determine:

- A hypothesis h in H such that h(x)=c(x) for all x in D.



Inductive Learning Hypothesis

- Any hypothesis found to approximate the target function well over the training examples, will also approximate the target function well over the unobserved examples.

Number of Instances, Concepts, Hypotheses

- Sky: Sunny, Cloudy, Rainy
- AirTemp: Warm, Cold
- Humidity: Normal, High
- Wind: Strong, Weak
- Water: Warm, Cold
- Forecast: Same, Change

#distinct instances : 3*2*2*2*2*2 = 96

#distinct concepts : 2^{96}

#syntactically distinct hypotheses : 5*4*4*4*4*4=5120

#semantically distinct hypotheses : 1+4*3*3*3*3*3=973

General to Specific Order

- Consider two hypotheses:
  - h_1=< Sunny,?,?,Strong,?,?>
  - h_2=< Sunny,?,?,?,?,?>
- Set of instances covered by h_1 and h_2:

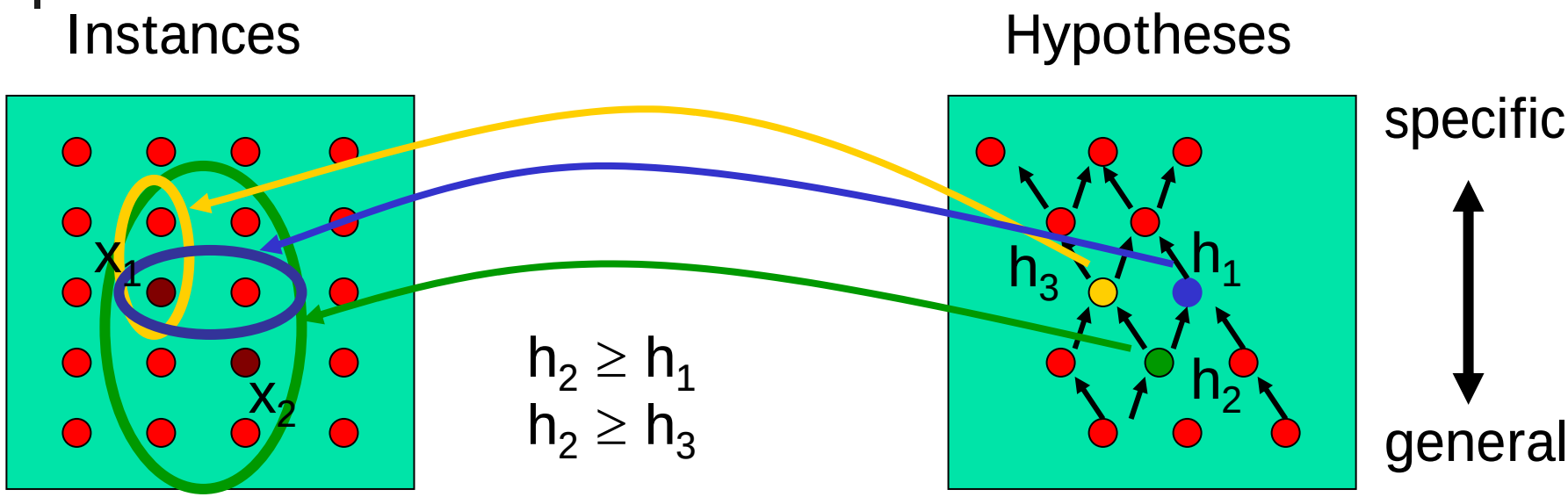
  h_2 imposes fewer constraints than h_1 and therefore classifies more instances x as positive h(x)=1.

Definition: Let h_j and h_k be boolean-valued functions defined over X. Then h_j is **more general than or equal to** h_k (written $h_j \geq h_k$) if and only if

$$\forall x \in X : [(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$

- The relation $\geq$ imposes a partial order over the hypothesis space H that is utilized many concept learning methods.

Instance, Hypotheses and "more general"



x_1=< Sunny,Warm,High,Strong,Cool,Same>

x_2=< Sunny,Warm,High,Light,Warm,Same>

h_1=< Sunny,?,?,Strong,?,?>

h_2=< Sunny,?,?,?,?,?>

h_3=< Sunny,?,?,?,Cool,?>

Find-S Algorithm

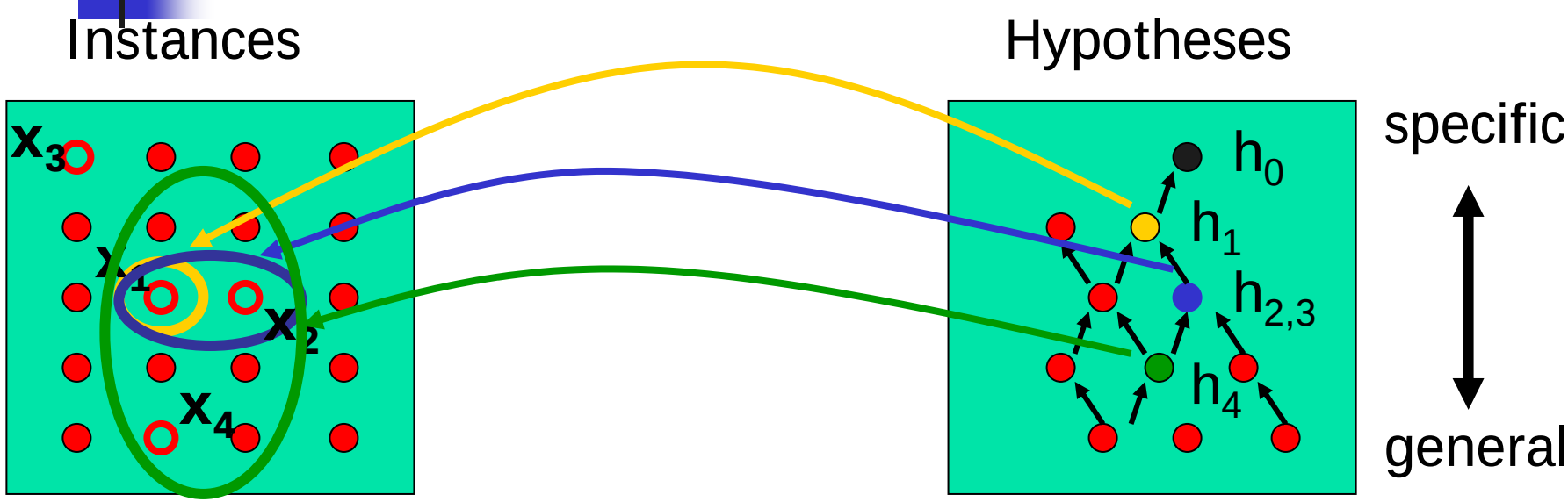
1. Initialize h to the most specific hypothesis in H
2. For each positive training instance x
   - For each attribute constraint a_i in h

     If the constraint a_i in h is satisfied by x

     then do nothing

     else replace a_i in h by the next more general constraint that is satisfied by x
3. Output hypothesis h

Hypothesis Space Search by Find-S



x_1 = <Sunny,Warm,Normal,Strong,Warm,Same> +

x_2 = <Sunny,Warm,High,Strong,Warm,Same> +

x_3 = <Rainy,Cold,High,Strong,Warm,Change> -

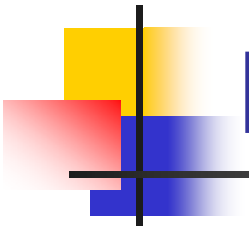
x_4 = <Sunny,Warm,High,Strong,Cool,Change> +

h_0 = < Ø, Ø, Ø, Ø, Ø, Ø,>

h_1 = < Sunny,Warm,Normal, Strong,Warm,Same>

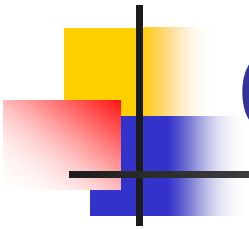
$h_{2,3}$ = < Sunny,Warm,?, Strong,Warm,Same>

h_4 = < Sunny,Warm,?, Strong,?,?>



Properties of Find-S

- Hypothesis space described by conjunctions of attributes
- Find-S will output the most specific hypothesis within H that is consistent with the positve training examples
- The output hypothesis will also be consistent with the negative examples, provided the target concept is contained in H.



Complaints about Find-S

- Can't tell if the learner has converged to the target concept, in the sense that it is unable to determine whether it has found the *only* hypothesis consistent with the training examples.
- Can't tell when training data is inconsistent, as it ignores negative training examples.
- Why prefer the most specific hypothesis?
- What if there are multiple maximally specific hypothesis?

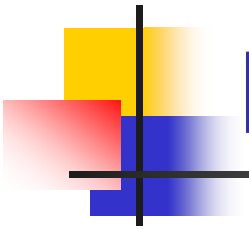
Version Spaces

- A hypothesis h is **consistent** with a set of training examples D of target concept if and only if h(x)=c(x) for each training example <x,c(x)> in D.

Consistent(h,D) := $\forall <x,c(x)> \in D \;\; h(x)=c(x)$

- The **version space**, $VS_{H,D}$, with respect to hypothesis space H, and training set D, is the subset of hypotheses from H consistent with all training examples:

$VS_{H,D} = \{h \in H \mid \text{Consistent}(h,D)\}$

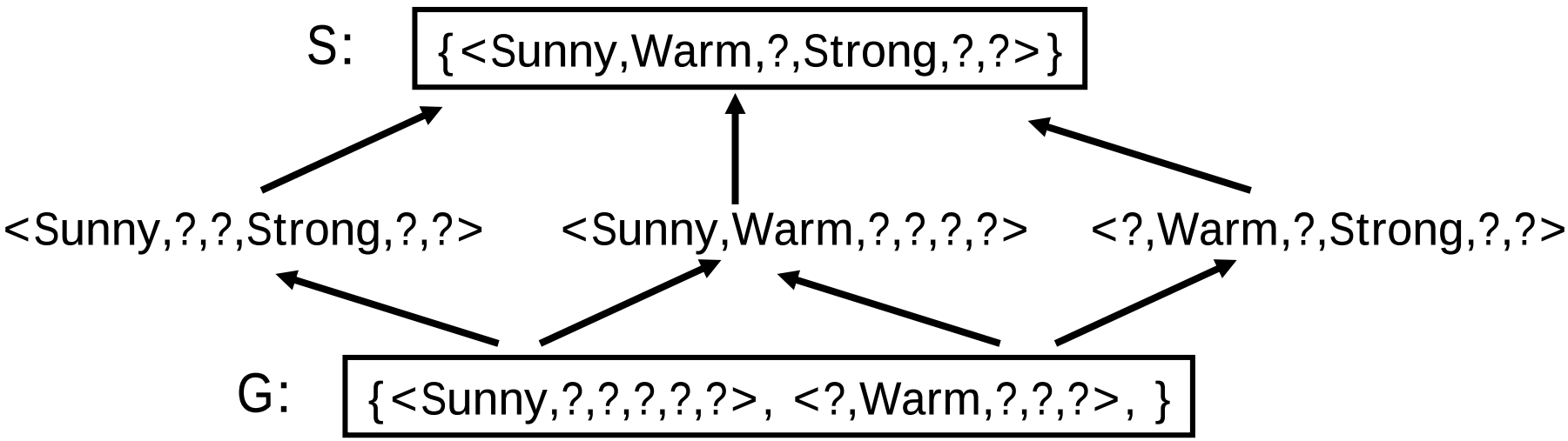


List-Then Eliminate Algorithm

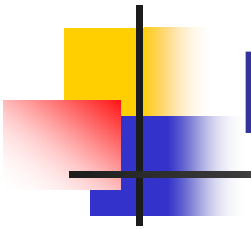
1. *VersionSpace* $\leftarrow$ a list containing every hypothesis in H
2. For each training example <x,c(x)>

   remove from *VersionSpace* any hypothesis that is inconsistent with the training example $h(x) \neq c(x)$
3. Output the list of hypotheses in *VersionSpace*

Example Version Space



x_1 = <Sunny Warm Normal Strong Warm Same> +
x_2 = <Sunny Warm High Strong Warm Same> +
x_3 = <Rainy Cold High Strong Warm Change> -
x_4 = <Sunny Warm High Strong Cool Change> +



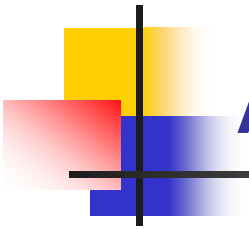
Representing Version Spaces

- The **general boundary**, G, of version space $VS_{H,D}$ is the set of maximally general members.
- The **specific boundary**, S, of version space $VS_{H,D}$ is the set of maximally specific members.
- Every member of the version space lies between these boundaries

$$VS_{H,D} = \{h \in H | (\exists\, s \in S)\ (\exists\, g \in G)\ (g \geq h \geq s)$$

where $x \geq y$ means x is more general or equal than y

Candidate Elimination Algorithm



$G \leftarrow$ maximally general hypotheses in H

S $\leftarrow$ maximally specific hypotheses in H

For each training example d=<x,c(x)>

If d is a positive example

Remove from G any hypothesis that is inconsistent with d

For each hypothesis s in S that is not consistent with d

- remove s from S.
- Add to S all minimal generalizations h of s such that
  - h consistent with d
  - Some member of G is more general than h
- Remove from S any hypothesis that is more general than another hypothesis in S

Candidate Elimination Algorithm (cont.)

If d is a negative example

Remove from S any hypothesis that is inconsistent with d

For each hypothesis g in G that is not consistent with d

- remove g from G.
- Add to G all minimal specializations h of g such that
  - h consistent with d
  - Some member of S is more specific than h
- Remove from G any hypothesis that is less general than another hypothesis in G

Example Candidate Elimination. Positive examples x_1 and x_2

S: {<∅, ∅, ∅, ∅, ∅, ∅ >}

G: {<?, ?, ?, ?, ?, ?>}

x_1 = <Sunny Warm Normal Strong Warm Same> +

S: {< Sunny Warm Normal Strong Warm Same >}

G: {<?, ?, ?, ?, ?, ?>}

x_2 = <Sunny Warm High Strong Warm Same> +

S: {< Sunny Warm ? Strong Warm Same >}

G: {<?, ?, ?, ?, ?, ?>}

Example Candidate Elimination. Negative and positive examples

S: {< Sunny Warm ? Strong Warm Same >}

G: {<?, ?, ?, ?, ?, ?>}

x_3 = <Rainy Cold High Strong Warm Change> -

S: {< Sunny Warm ? Strong Warm Same >}

G: {<Sunny,?,?,?,?,?>, <?,Warm,?,?,?>, <?,?,?,?,?,Same>}

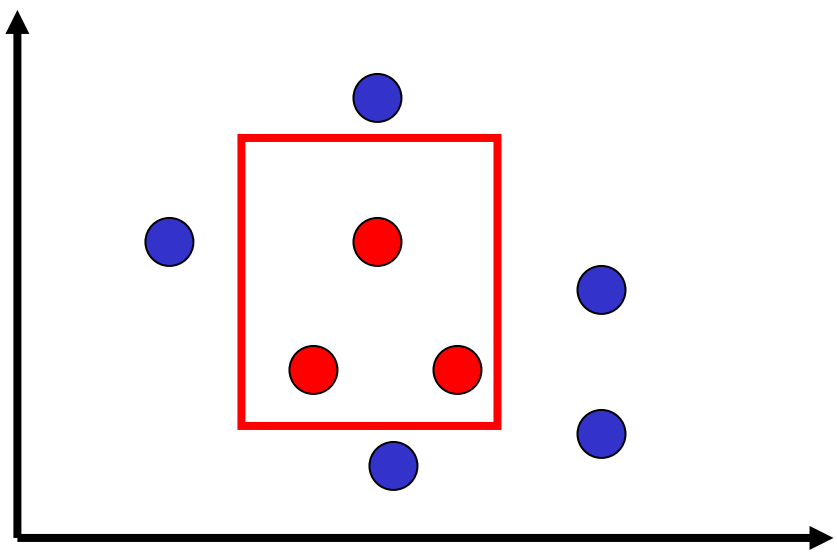
x_4 = <Sunny Warm High Strong Cool Change> +

S: {< Sunny Warm ? Strong ? ? >}

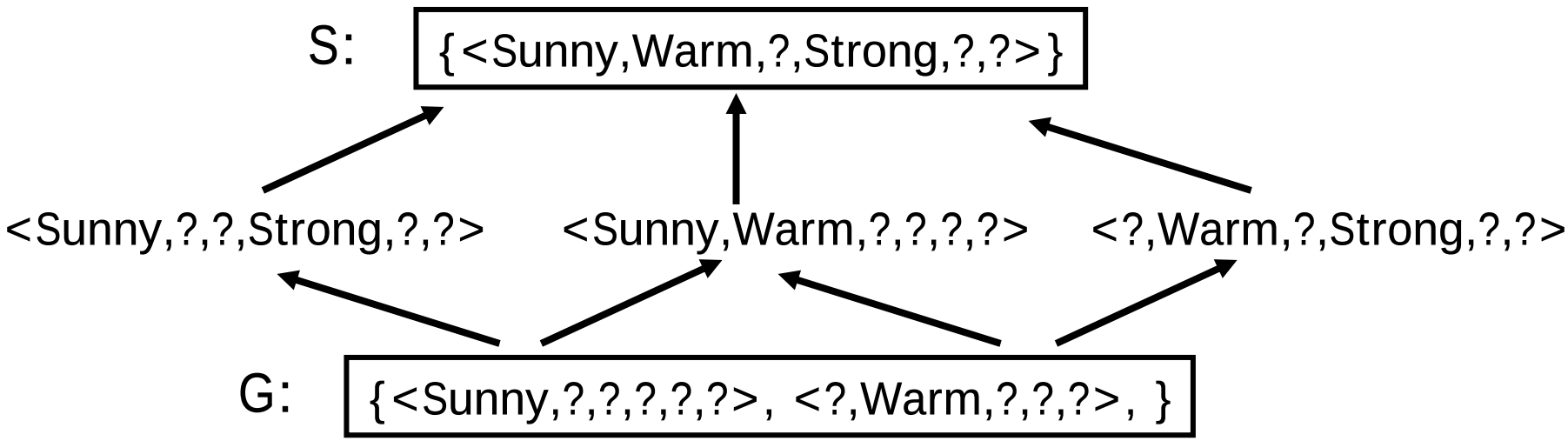
G: {<Sunny,?,?,?,?,?>, <?,Warm,?,?,?> }

Example Candidate Elimination. Geometric interpretation

- Instance space: integer points in the x,y plane
- hypothesis space : rectangles, that means hypotheses are of the form $a \leq x \leq b$, $c \leq y \leq d$.



Classification of New Data

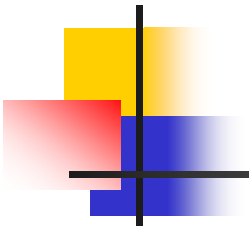


x_5 = <Sunny Warm Normal Strong Cool Change> + 6/0

x_6 = <Rainy Cold Normal Light Warm Same> - 0/6

x_7 = <Sunny Warm Normal Light Warm Same> ? 3/3

x_8 = <Sunny Cold Normal Strong Warm Same> ? 2/4



Inductive Leap

+ <Sunny Warm Normal Strong Cool Change>

+ <Sunny Warm Normal Light Warm Same>

S : <Sunny Warm Normal ? ? ?>

- How can we justify to classify the new example as

  + <Sunny Warm Normal Strong Warm Same>

Bias: We assume that the hypothesis space H contains the target concept c. In other words that c can be described by a conjunction of literals.

Biased Hypothesis Space

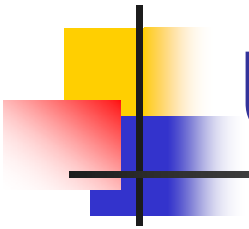
- Our hypothesis space is unable to represent a simple disjunctive target concept : (Sky=Sunny) v (Sky=Cloudy)

x_1 = <Sunny Warm Normal Strong Cool Change> +
x_2 = <Cloudy Warm Normal Strong Cool Change> +

S : { <?, Warm, Normal, Strong, Cool, Change> }

x_3 = <Rainy Warm Normal Light Warm Same> -

S : {}



Unbiased Learner

- Idea: Choose H that expresses every teachable concept, that means H is the set of all possible subsets of X called the power set P(X)
- $|X|=96$, $|P(X)|=2^{96} \sim 10^{28}$ distinct concepts
- H = disjunctions, conjunctions, negations
  - e.g. <Sunny Warm Normal ? ? ?> v <? ? ? ? ? Change>
- H surely contains the target concept.

Unbiased Learner (cont.)

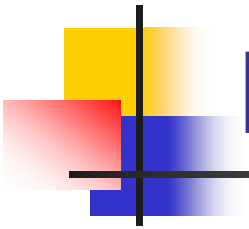
What are S and G in this case?

Assume positive examples (x_1, x_2, x_3) and negative examples (x_4, x_5)

S : { $(x_1 \vee x_2 \vee x_3)$ } G : { $\neg (x_4 \vee x_5)$ }

The only examples that are classified are the training examples themselves. In other words in order to learn the target concept one would have to present every single instance in X as a training example.

Each unobserved instance will be classified positive by precisely half the hypothesis in VS and negative by the other half.



Futility of Bias-Free Learning

- A learner that makes no prior assumptions regarding the identity of the target concept has no rational basis for classifying any unseen instances.

Inductive Bias

Consider:

- Concept learning algorithm L
- Instances X, target concept c
- Training examples $D_c = \{<x, c(x)>\}$
- Let $L(x_i, D_c)$ denote the classification assigned to instance x_i by L after training on D_c.

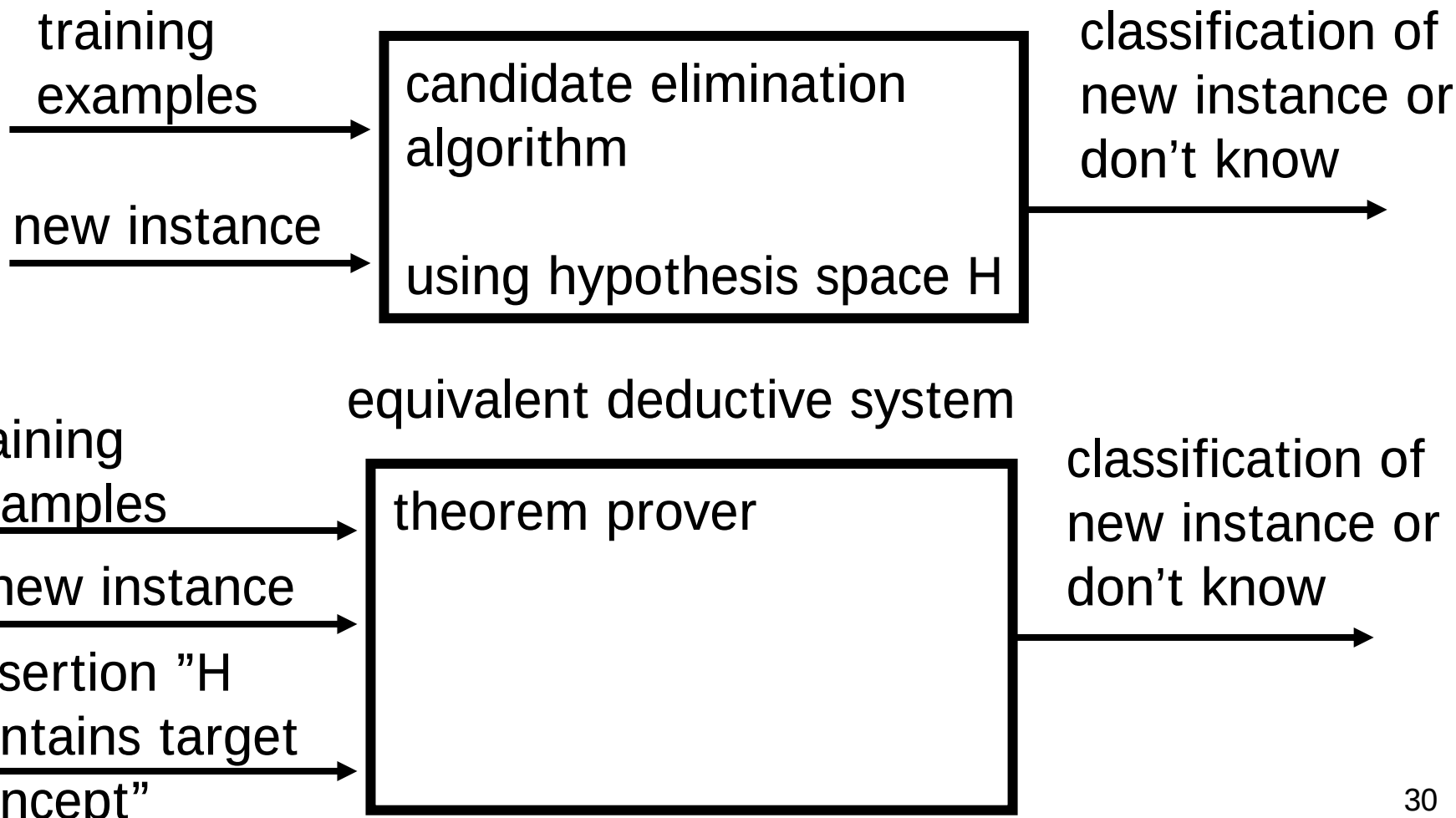
Definition:

The inductive bias of L is any minimal set of assertions B such that for any target concept c and corresponding training data D_c

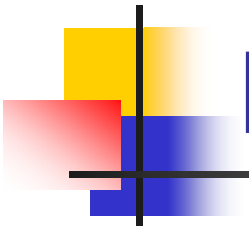
$$(\forall x_i \in X)[B \wedge D_c \wedge x_i] \vdash L(x_i, D_c)$$

Where A |-- B means that A logically entails B.

Inductive Systems and Equivalent Deductive Systems



Three Learners with Different Biases



- Rote learner: Store examples classify x if and only if it matches a previously observed example.
  - No inductive bias
- Version space candidate elimination algorithm.
  - Bias: The hypothesis space contains the target concept.
- Find-S
  - Bias: The hypothesis space contains the target concept and all instances are negative instances unless the opposite is entailed by its other knowledge.