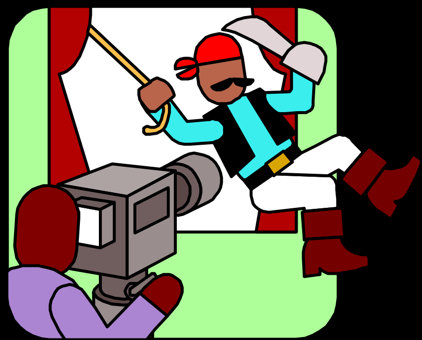
CSC I6716
Spring 2004



Topic 3 of Part 1
Image Formation

http://www-cs.engr.ccny.cuny.edu/~zhu/VisionCourse-I6716.html

Acknowledgements

The slides in this lecture were adopted from

Professor Allen Hanson
University of Massachusetts at Amherst

Lecture Outline

- Light and Optics
  - Pinhole camera model
  - Perspective projection
  - Thin lens model
  - Fundamental equation
  - Distortion: spherical & chromatic aberration, radial distortion (*optional)
  - Reflection and Illumination: color, lambertian and specular surfaces, Phong, BDRF (*optional)
- Sensing Light
- Conversion to Digital Images
- Sampling Theorem
- Other Sensors: frequency, type, ….

Abstract Image

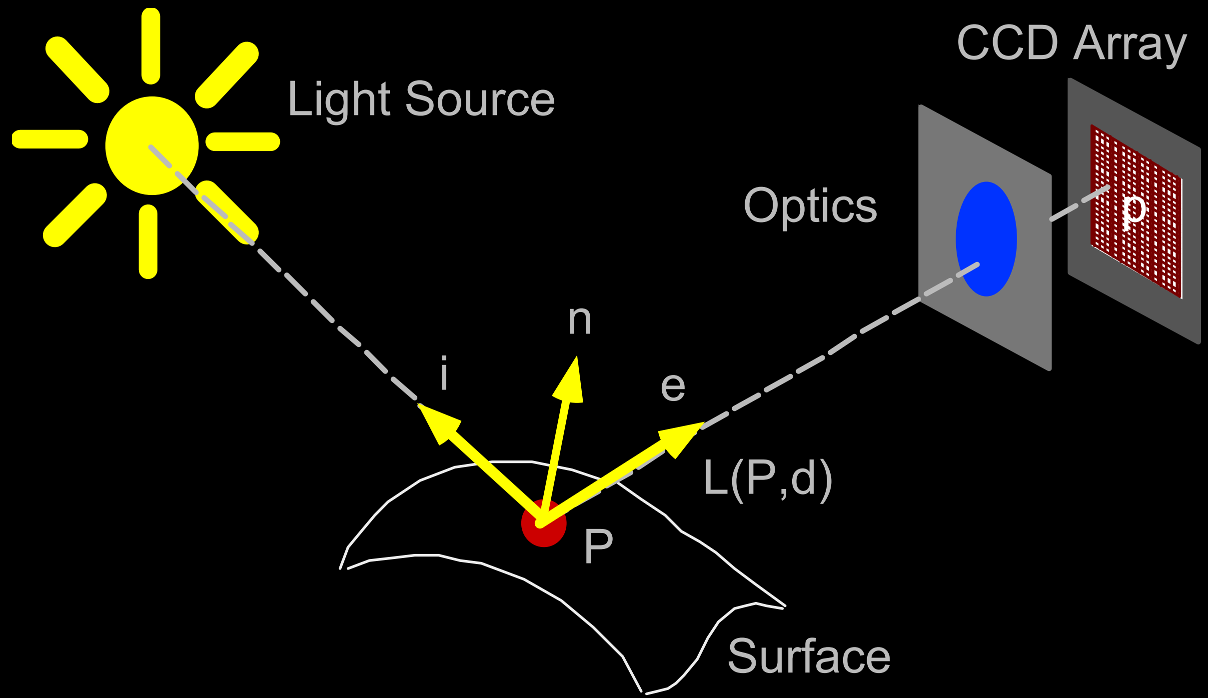
- An image can be represented by an image function whose general form is **f(x,y)**.
- **f(x,y)** is a vector-valued function whose arguments represent a pixel location.
- The value of **f(x,y)** can have different interpretations in different kinds of images.

Examples

Intensity Image	- f(x,y) = intensity of the scene
Range Image	- f(x,y) = depth of the scene from imaging system
Color Image	- **f**(x,y) = $\{f_r(x,y), f_g(x,y), f_b(x,y)\}$
Video	- f(x,y,t) = temporal image sequence

Basic Radiometry

- Radiometry is the part of image formation concerned with the relation among the amounts of light energy emitted from light sources, reflected from surfaces, and registered by sensors.



Light and Matter

- The interaction between light and matter can take many forms:
  - Reflection
  - Refraction
  - Diffraction
  - Absorption
  - Scattering

Lecture Assumptions

- Typical imaging scenario:
  - visible light
  - ideal lenses
  - standard sensor (e.g. TV camera)
  - opaque objects
- Goal

> To create 'digital' images which can be processed to recover some of the characteristics of the 3D world which was imaged.

Steps

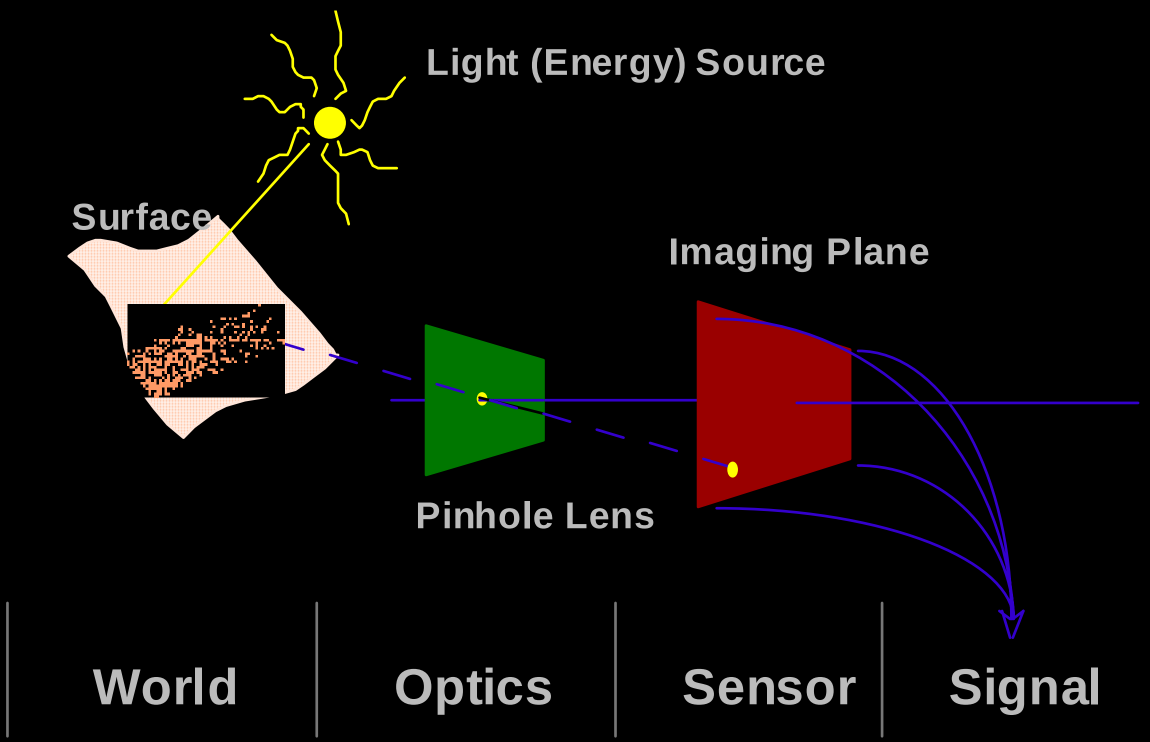
World → Optics → Sensor →
Signal → Digitizer →
Digital Representation

World	reality
Optics	focus {light} from world on sensor
Sensor	converts {light} to {electrical energy}
Signal	representation of incident light as continuous electrical energy
Digitizer	converts continuous signal to discrete signal
Digital Rep.	final representation of reality in computer memory

Factors in Image Formation

- Geometry
  - concerned with the relationship between points in the three-dimensional world and their images
- Radiometry
  - concerned with the relationship between the amount of light radiating from a surface and the amount incident at its image
- Photometry
  - concerned with ways of measuring the intensity of light
- Digitization
  - concerned with ways of converting continuous signals (in both space and time) to digital approximations

Image Formation



Sensor	Signal
B&W Film	Silver Density
Color Film	Silver density in three color layers
TV Camera	Electrical

- Geometry describes the projection of:

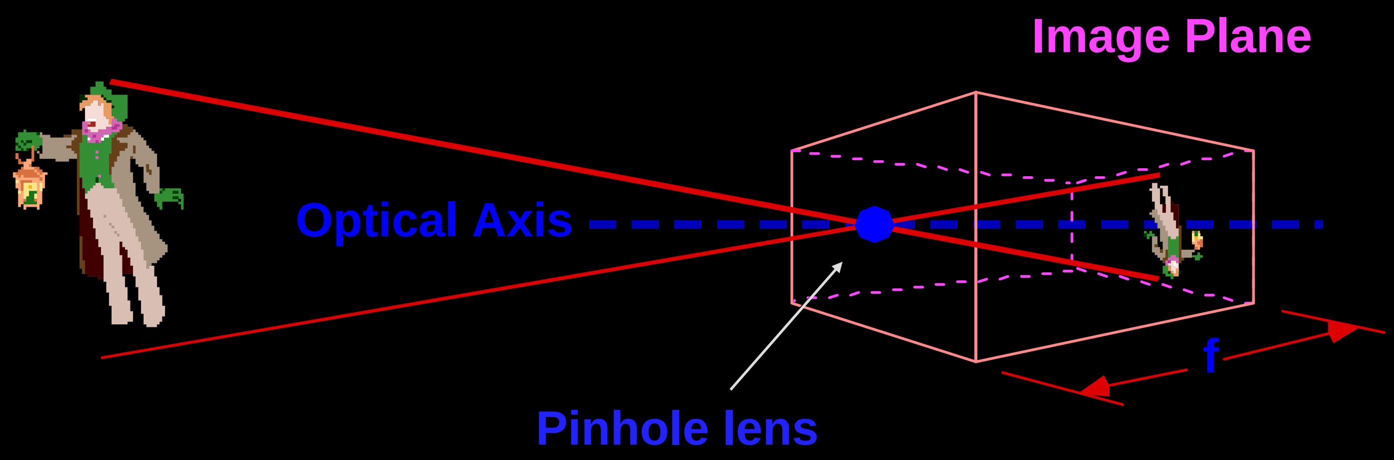
three-dimensional (3D) world → two-dimensional (2D) image plane.

- Typical Assumptions
  - Light travels in a straight line
- **Optical Axis**: the axis perpendicular to the image plane and passing through the pinhole (also called the central projection ray)
- Each point in the image corresponds to a particular direction defined by a **ray** from that point through the pinhole.
- Various kinds of projections:
  - - perspective - oblique
  - - orthographic - isometric
  - - spherical

Basic Optics

- Two models are commonly used:
  - Pin-hole camera
  - Optical system composed of lenses
- **Pin-hole** is the basis for most graphics and vision
  - Derived from physical construction of early cameras
  - Mathematics is very straightforward
- **Thin lens** model is first of the lens models
  - Mathematical model for a physical lens
  - Lens gathers light over area and focuses on image plane.

Pinhole Camera Model



- World projected to 2D Image
  - Image inverted
  - Size reduced
  - Image is dim
  - No direct depth information
- f called the focal length of the lens
- Known as perspective projection

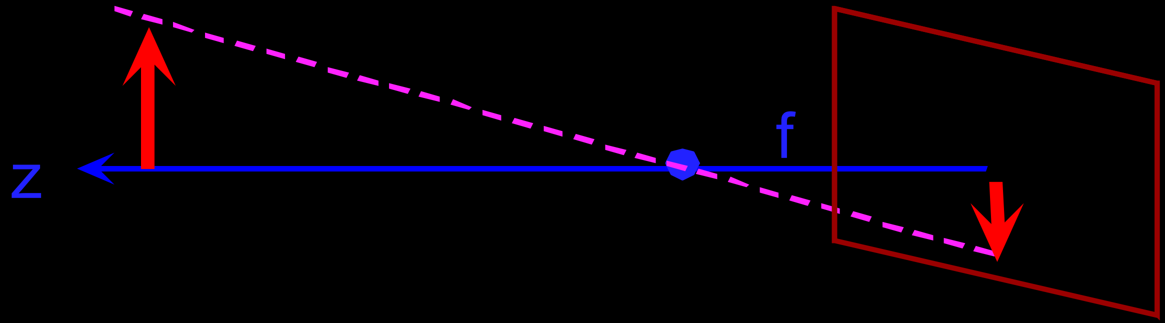
Pinhole camera image

Amsterdam

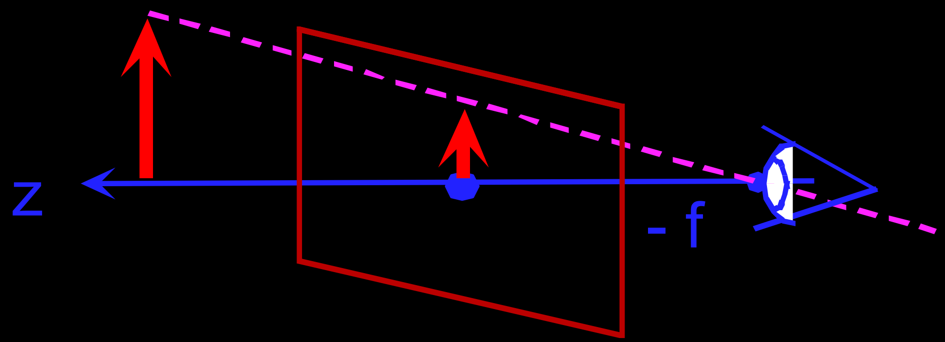
Photo by Robert Kosara, robert@kosara.net
http://www.kosara.net/gallery/pinholeamsterdam/pic01.html

Equivalent Geometry

- Consider case with object on the optical axis:



- More convenient with upright image:

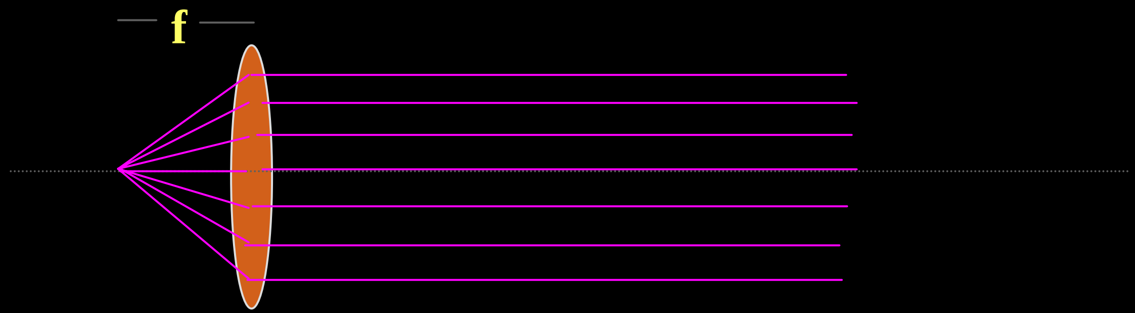


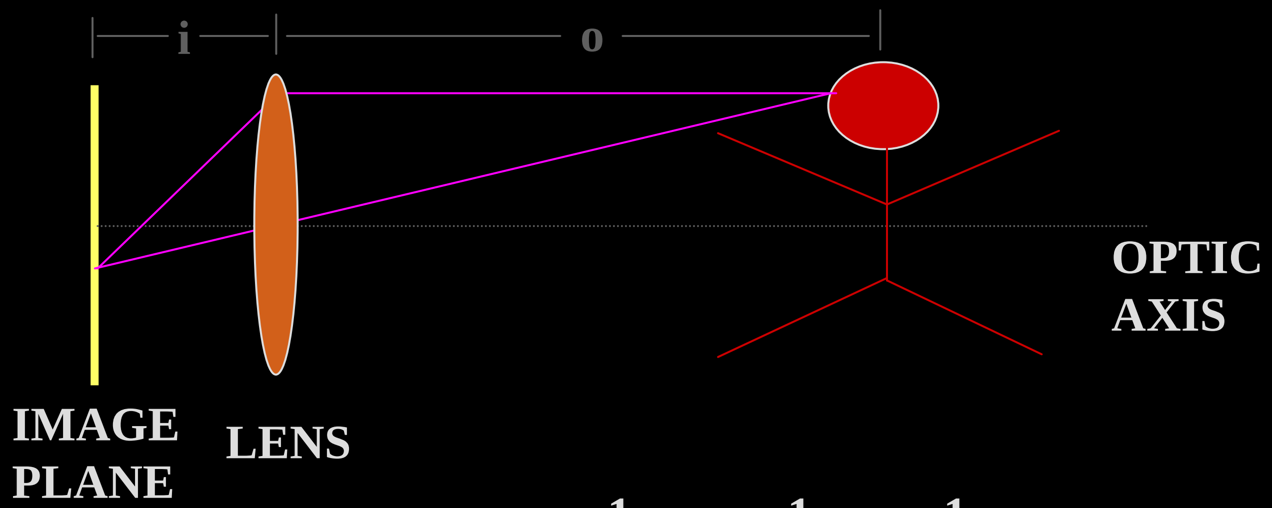
Projection plane z = 0

- Equivalent mathematically

Thin Lens Model

- Rays entering parallel on one side converge at focal point.
- Rays diverging from the focal point become parallel.

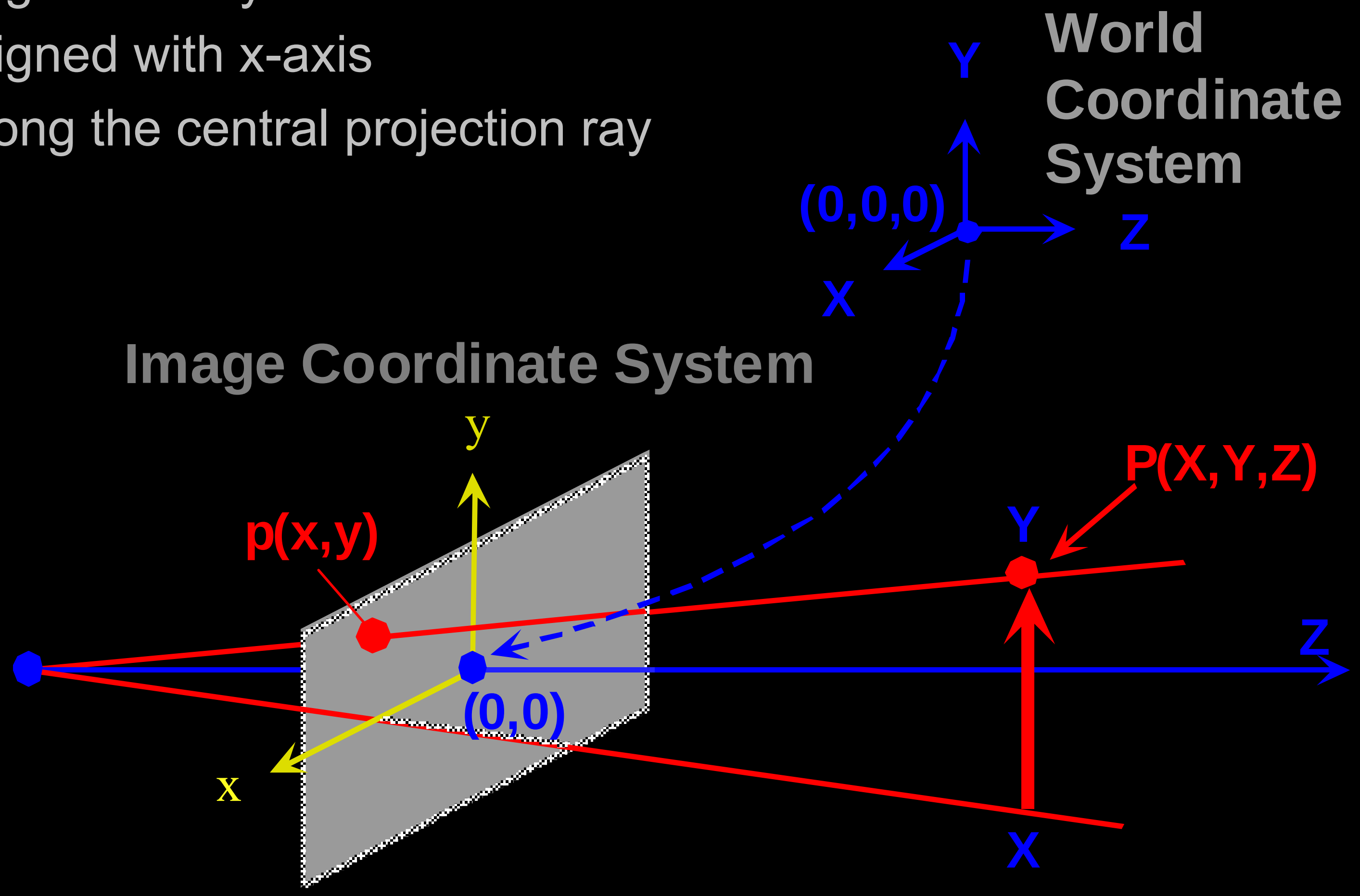




$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$ 'THIN LENS LAW'

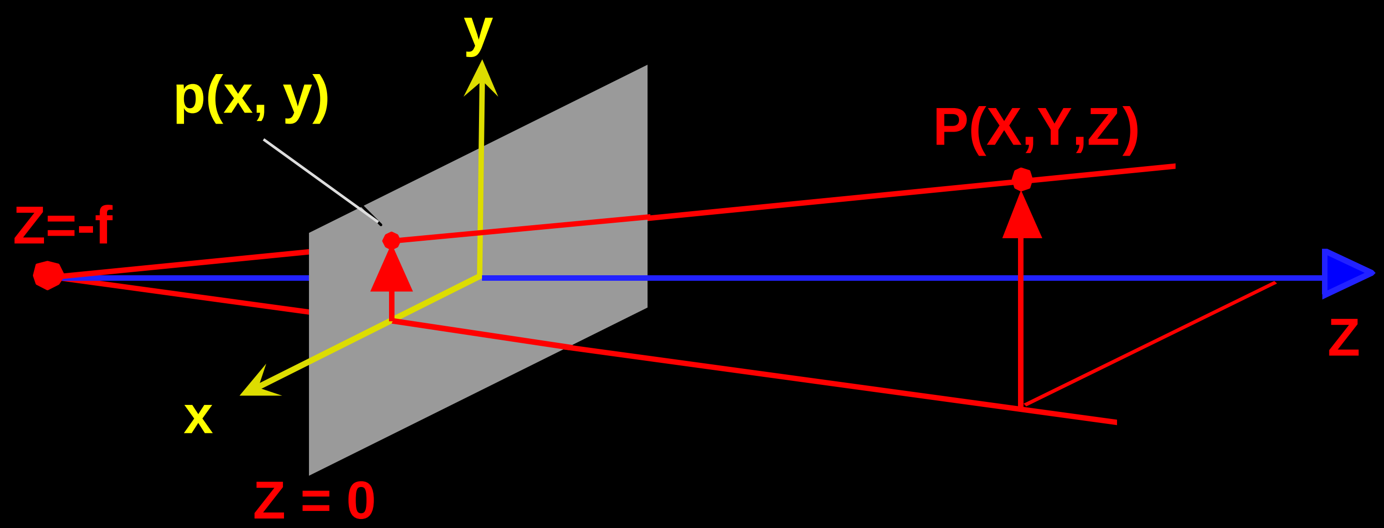
Coordinate System

- Simplified Case:
  - Origin of world and image coordinate systems coincide
  - Y-axis aligned with y-axis
  - X-axis aligned with x-axis
  - Z-axis along the central projection ray



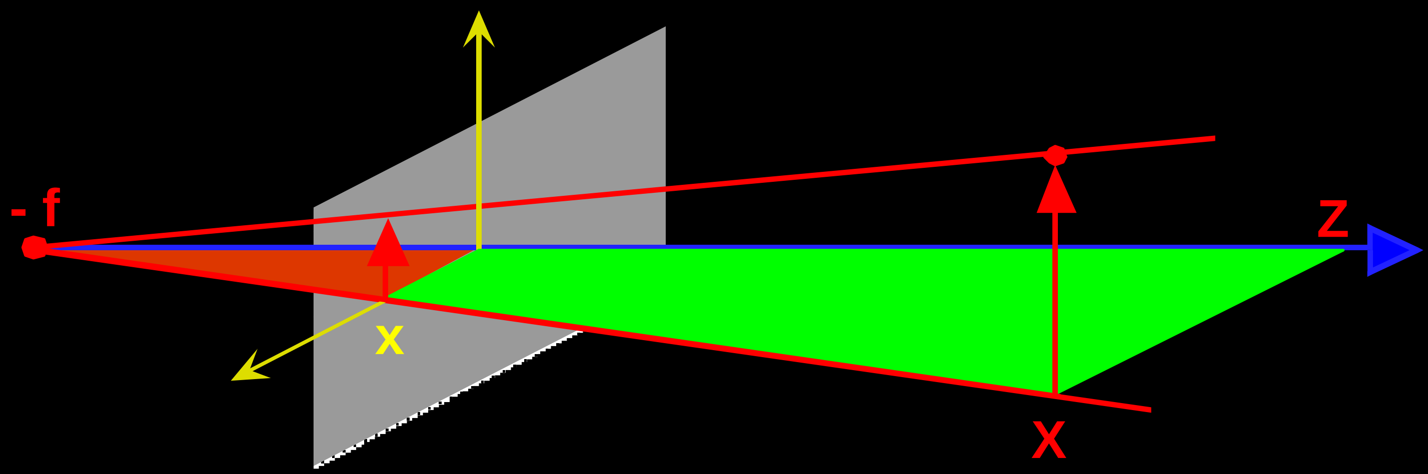
Perspective Projection

- Compute the image coordinates of p in terms of the world coordinates of P.



- Look at projections in x-z and y-z planes

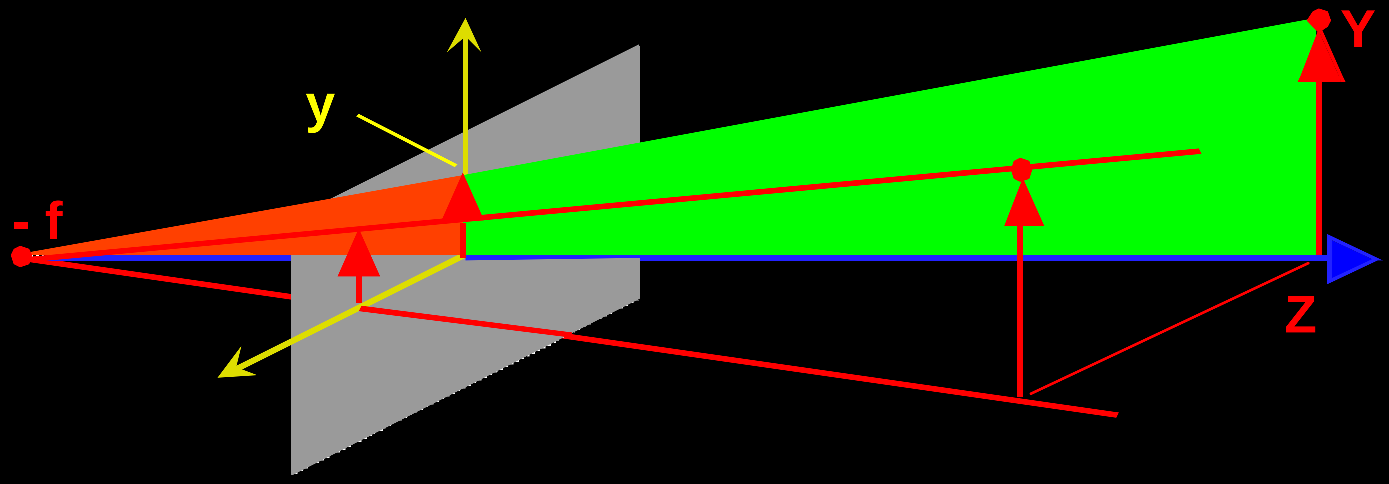
X-Z Projection



- By similar triangles: $\frac{x}{f} = \frac{X}{Z+f}$

$$x = \frac{fX}{Z+f}$$

Y-Z Projection



- By similar triangles: $\frac{y}{f} = \frac{Y}{Z+f}$

$$y = \frac{fY}{Z+f}$$

Perspective Equations

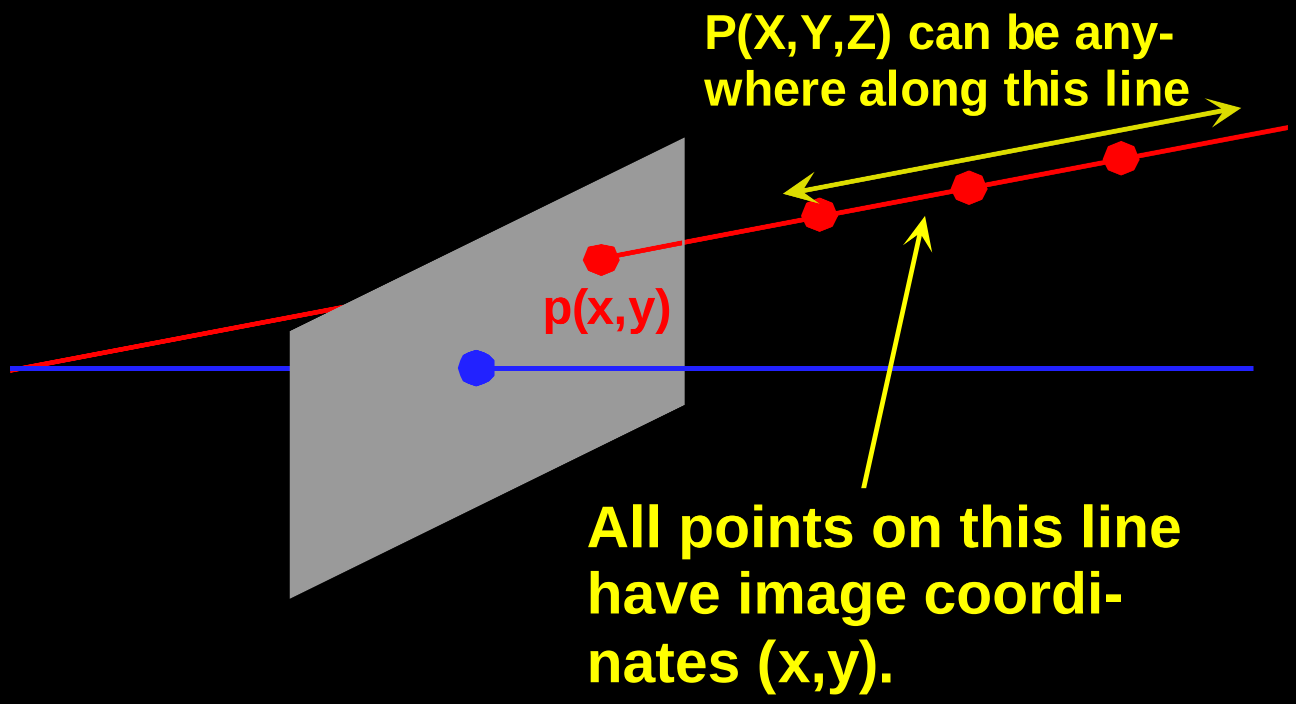
- Given point P(X,Y,Z) in the 3D world
- The two equations:

$$x = \frac{fX}{Z+f} \qquad y = \frac{fY}{Z+f}$$

- transform world coordinates (X,Y,Z) into image coordinates (x,y)
- Question:
  - What is the equation if we select the origin of both coordinate systems at the nodal point?

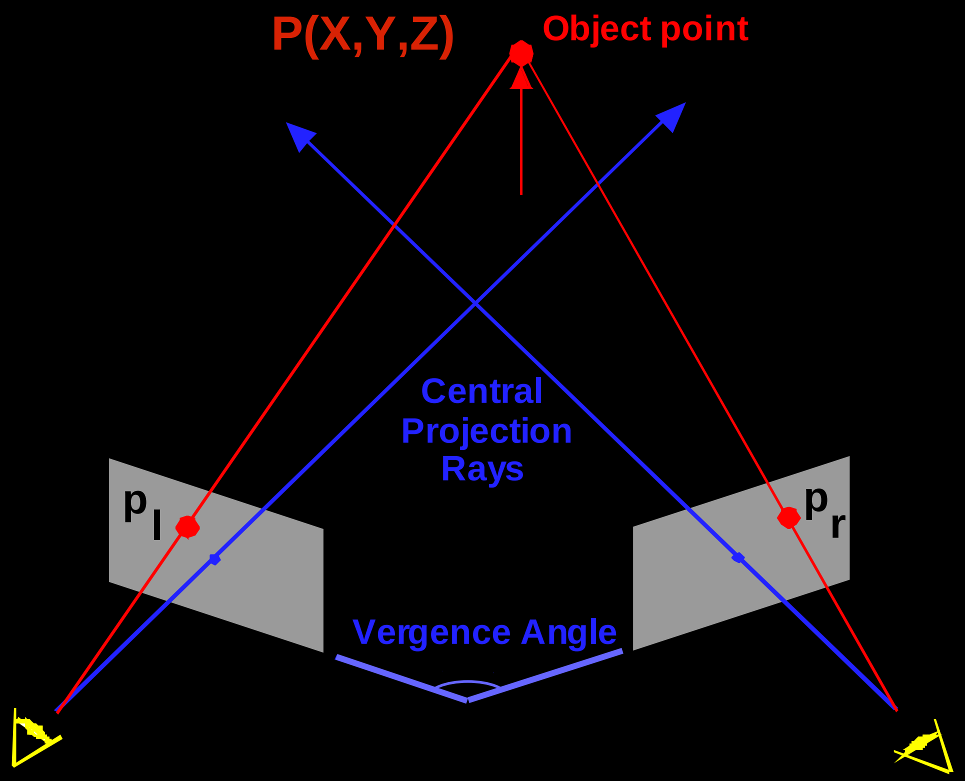
Reverse Projection

- Given a center of projection and image coordinates of a point, it is not possible to recover the 3D depth of the point from a single image.



In general, at least two images of the same point taken from two different locations are required to recover depth.

Stereo Geometry



- Depth obtained by triangulation
- Correspondence problem: p_l and p_r must correspond to the left and right projections of P, respectively.

Radiometry

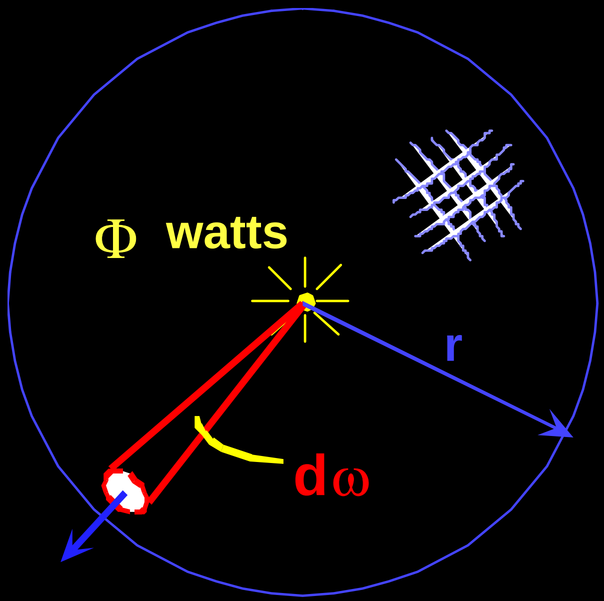
- Image: two-dimensional array of 'brightness' values.
- Geometry: where in an image a point will project.
- Radiometry: what the brightness of the point will be.
- Brightness: informal notion used to describe both scene and image brightness.
- Image brightness: related to energy flux incident on the image plane: =>

  IRRADIANCE
- Scene brightness: brightness related to energy flux emitted (radiated) from a surface: =>

  RADIANCE

Light

- Electromagnetic energy
- Wave model
- Light sources typically radiate over a frequency spectrum
- Φ watts radiated into 4π radians



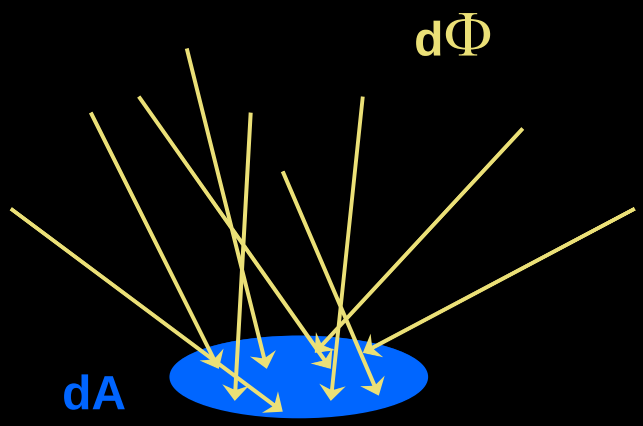
$$\Phi = \int_{sphere} d\Phi$$

R = Radiant Intensity = $\frac{d\Phi}{d\omega}$ Watts/unit solid angle (steradian)

(of source)

Irradiance

- Light falling on a surface from all directions.
- How much?

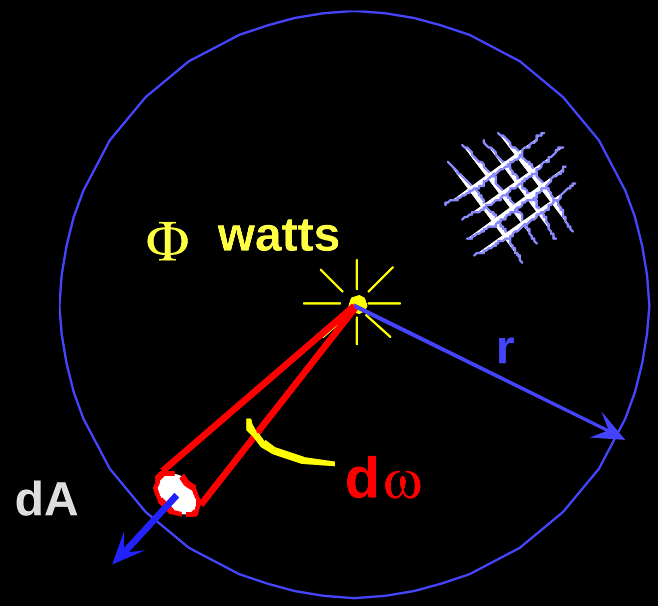


- Irradiance: power per unit area falling on a surface.

$$\text{Irradiance } E = \frac{d\Phi}{dA} \quad \text{watts/m}^2$$

Inverse Square Law

- Relationship between radiance (radiant intensity) and irradiance



$$d\omega = \frac{dA}{r^2}$$

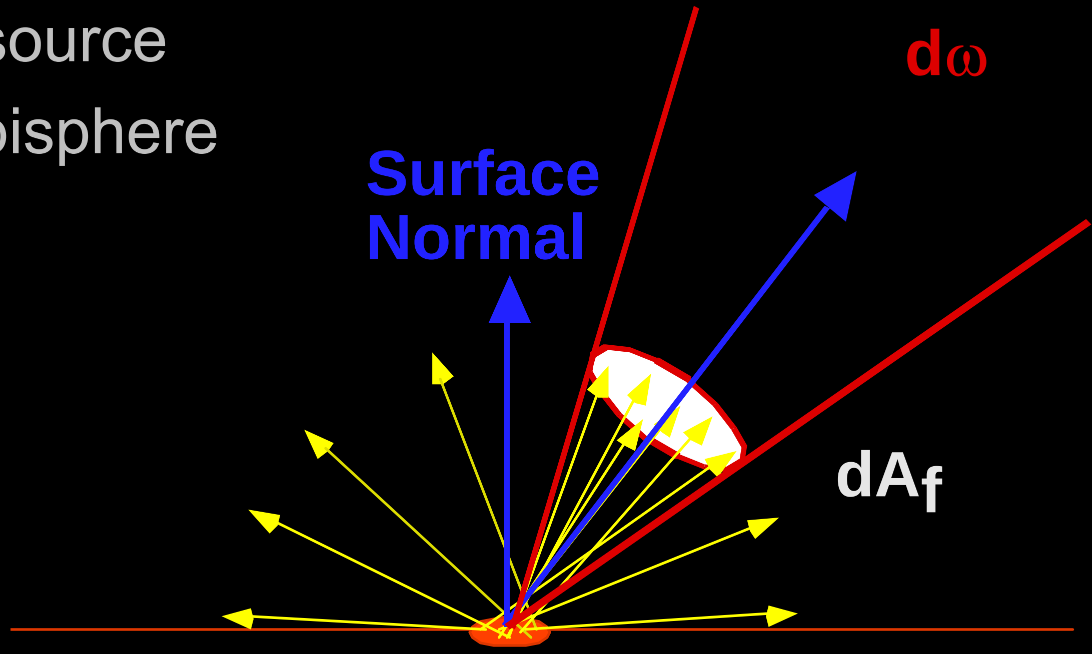
$$E = \frac{d\Phi}{dA}$$

$$R = \frac{d\Phi}{d\omega} = \frac{r^2 \, d\Phi}{dA} = r^2 E$$

$$E = \frac{R}{r^2}$$

Surface Radiance

- Surface acts as light source
- Radiates over a hempisphere

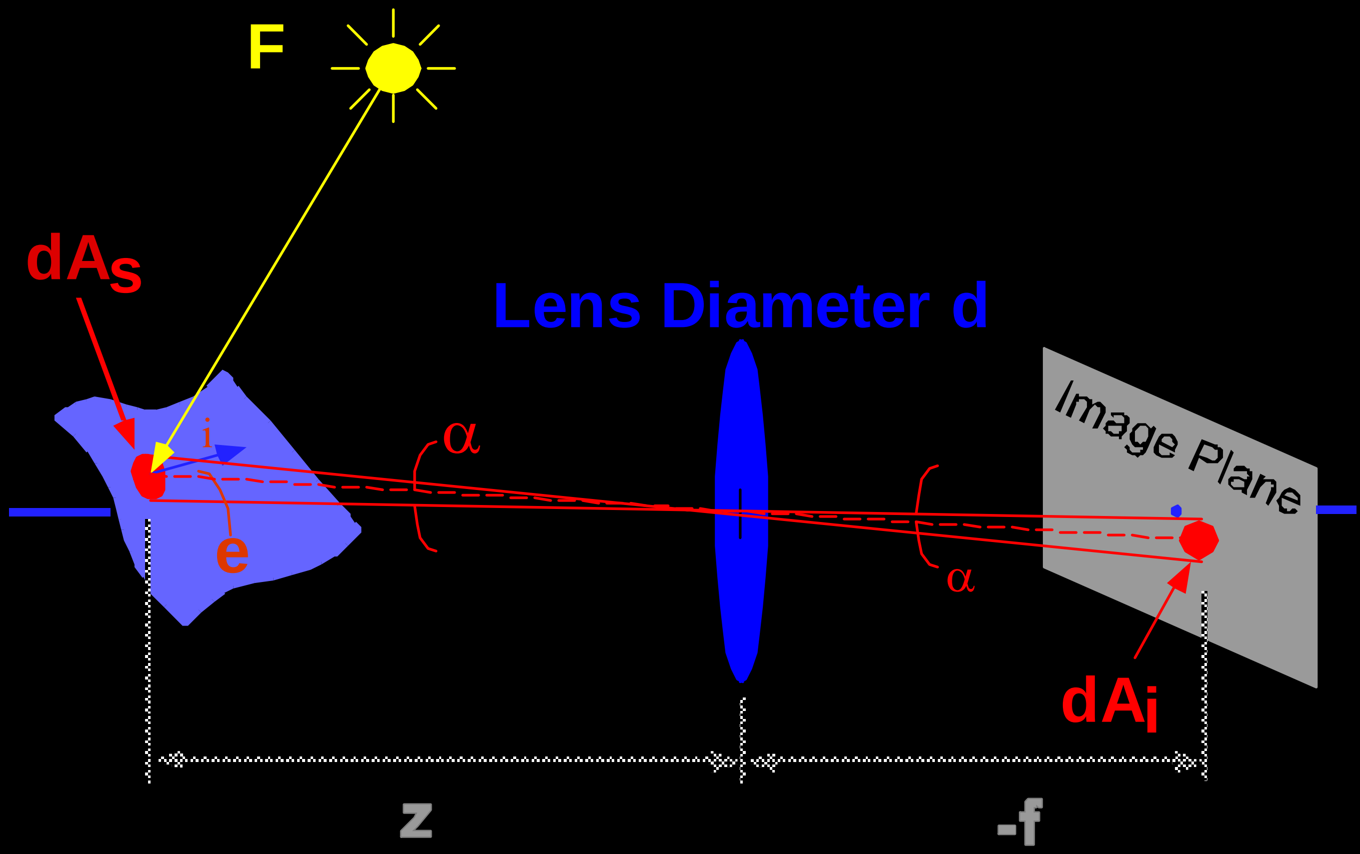


- Radiance: power per unit forshortened area emitted into a solid angle

$$L = \frac{d^2\Phi}{dA_f \, d\omega}$$

(watts/m2 - steradian)

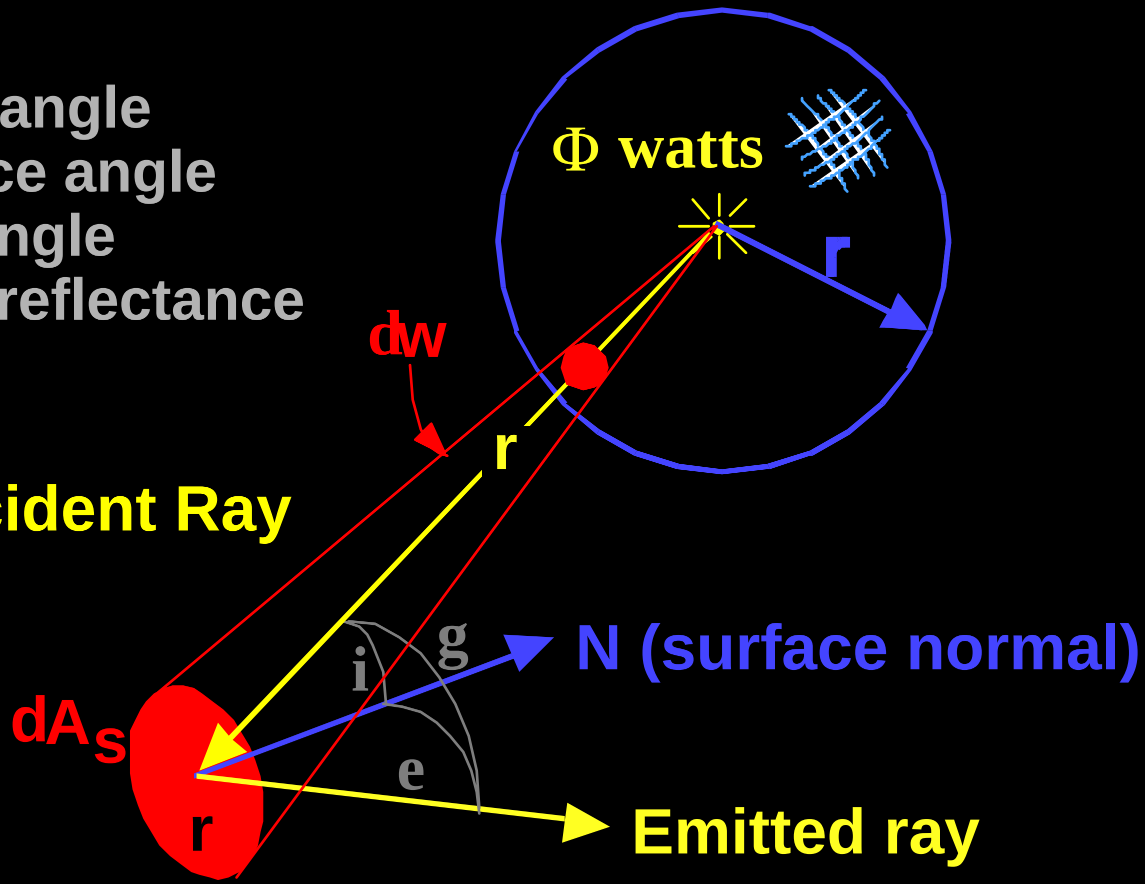
- Goal: Relate the radiance of a surface to the irradiance in the image plane of a simple optical system.



Light at the Surface

- E = flux incident on the surface (irradiance) = $\frac{d\Phi}{dA}$

i = incident angle
e = emittance angle
g = phase angle
r = surface reflectance



- We need to determine dΦ and dA

Reflections from a Surface I

dA

- dA = $dA_s \cos i$ {foreshortening effect in direction of light source}

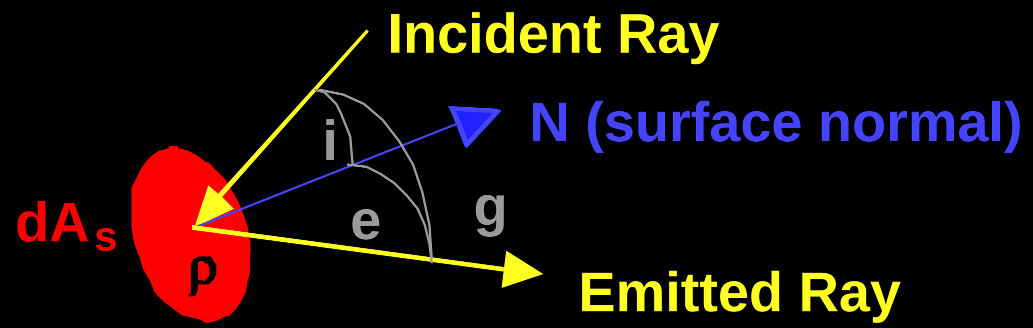
$d\Phi$

- $d\Phi$ = flux intercepted by surface over area dA
  - dA subtends solid angle $d\omega = dA_s \cos i / r^2$
  - $d\Phi = R\, d\omega = R\, dA_s \cos i / r^2$
  - $E = d\Phi / dA_s$

Surface Irradiance: $E = R \cos i / r^2$

Reflections from a Surface II

- Now treat small surface area as an emitter
  - ….because it is bouncing light into the world
- How much light gets reflected?

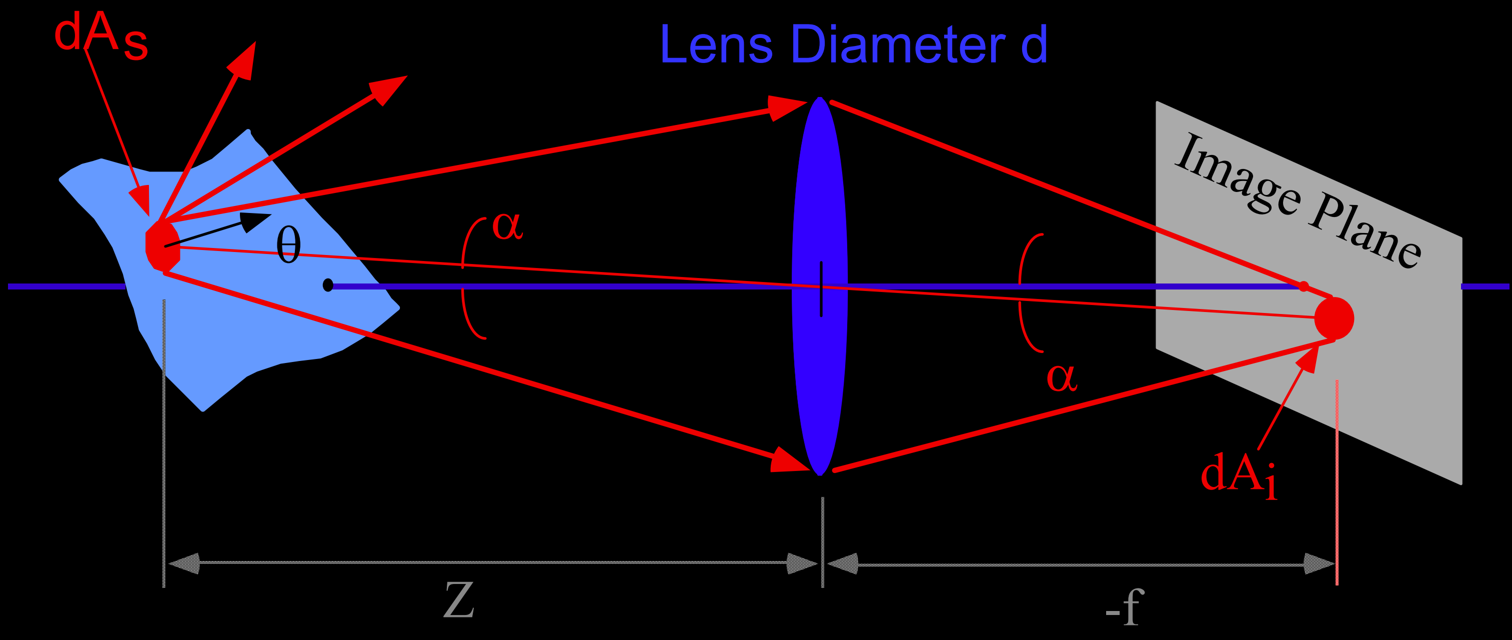


- E is the surface irradiance
- L is the surface radiance = luminance
- They are related through the surface reflectance function:

$$\frac{L_s}{E} = \rho(i,e,g,\lambda)$$

May also be a function of the wavelength of the light

Power Concentrated in Lens

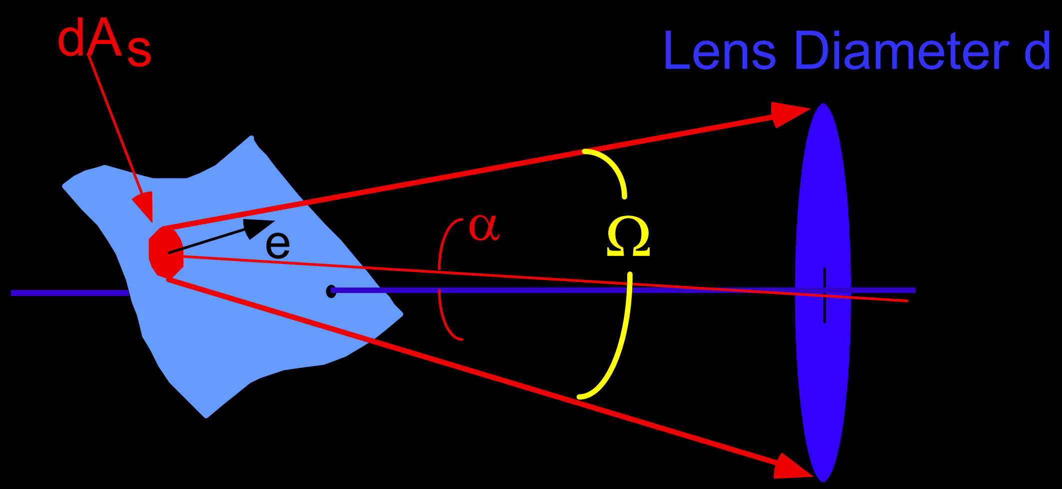


$$L_s = \frac{d^2\Phi}{dA_s d\omega}$$ Luminance of patch (known from previous step)

What is the power of the surface patch as a source in the direction of the lens?

$$d^2\Phi = L_s dA_s d\omega$$

Through a Lens Darkly



- In general:
  - L_s is a function of the angles i and e.
  - Lens can be quite large
  - Hence, must integrate over the lens solid angle to get $d\Phi$

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

Simplifying Assumption

- Lens diameter is small relative to distance from patch

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

L_s is a constant and can be removed from the integral

$$d\Phi = dA_s L_s \int_{\Omega} d\Omega$$

Surface area of patch in direction of lens

$= dA_s \cos e$

Solid angle subtended by lens in direction of patch

$$= \frac{\text{Area of lens as seen from patch}}{(\text{Distance from lens to patch})^2}$$

$$= \frac{\pi (d/2)^2 \cos \alpha}{(z / \cos \alpha)^2}$$

Putting it Together

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

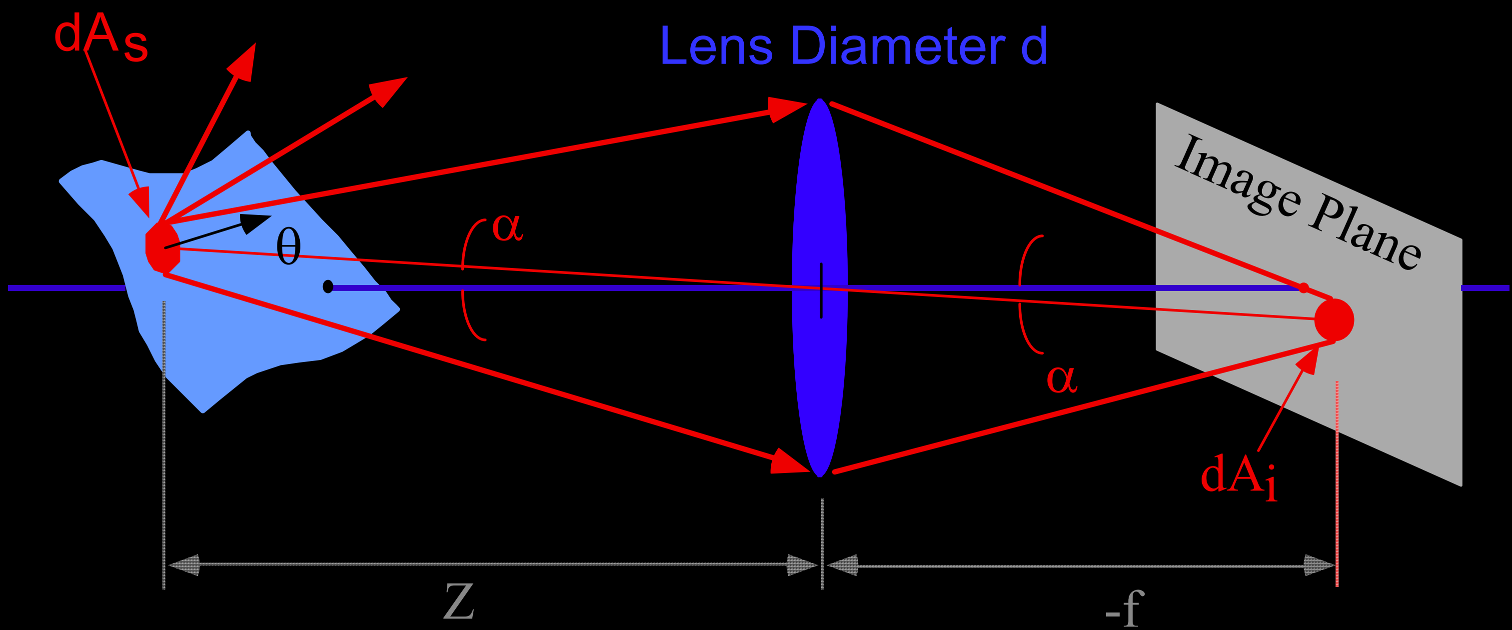
$$= dA_s \cos e \; L_s \frac{\pi \, (d/2)^2 \cos \alpha}{(z / \cos \alpha)^2}$$

- Power concentrated in lens:

$$d\Phi = \frac{\pi}{4} L_s \, dA_s \left[\frac{d}{z}\right]^2 \cos e \cos^3 \alpha$$

- Assuming a lossless lens, this is also the power radiated by the lens as a source.

Through a Lens Darkly

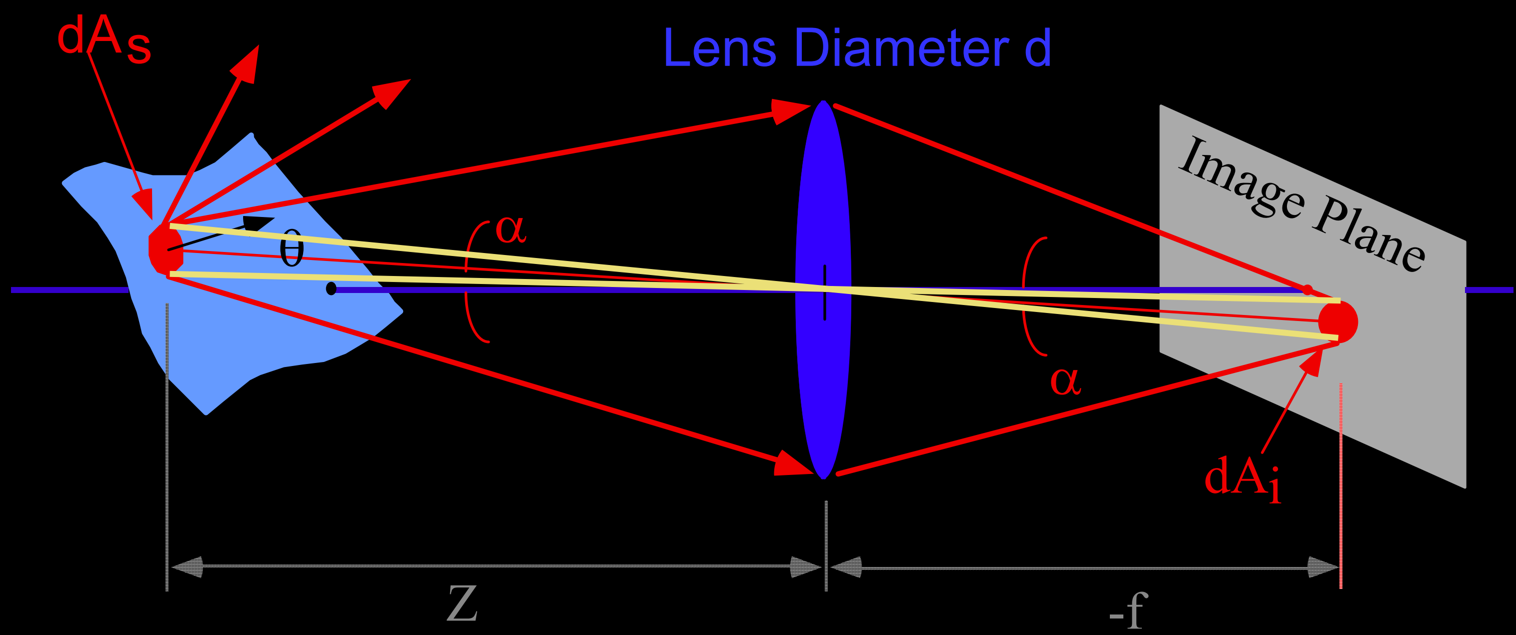


- Image irradiance at $dA_i = \frac{d\Phi}{dA_i} = E_i$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4} \left[\frac{d}{Z} \right]^2 \cos e \cos^3 \alpha$$

ratio of areas

Patch ratio



The two solid angles are equal

$$\frac{dA_s \cos e}{(Z / \cos \alpha)^2} = \frac{dA_i \cos \alpha}{(-f / \cos \alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos e}\left(\frac{Z}{-f}\right)^2$$

The Fundamental Result

- Source Radiance to Image Sensor Irradiance:

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos e}\left(\frac{Z}{-f}\right)^2$$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3\alpha$$

$$E_i = L_s \frac{\cos\alpha}{\cos e}\left(\frac{Z}{-f}\right)^2 \frac{\pi}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3\alpha$$

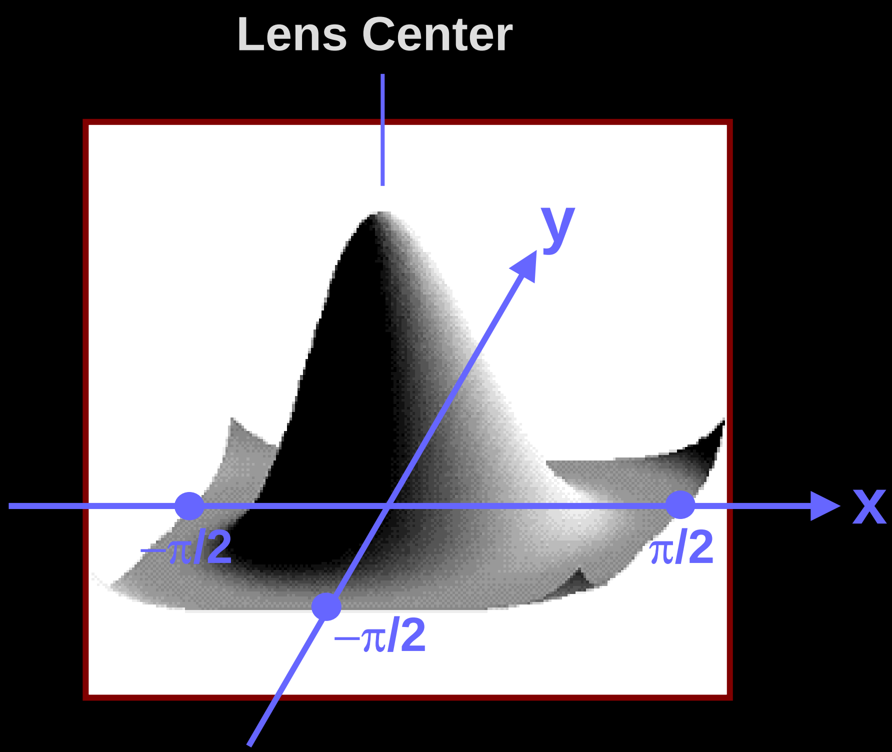
$$E_i = L_s \frac{\pi}{4}\left(\frac{d}{-f}\right)^2 \cos^4\alpha$$

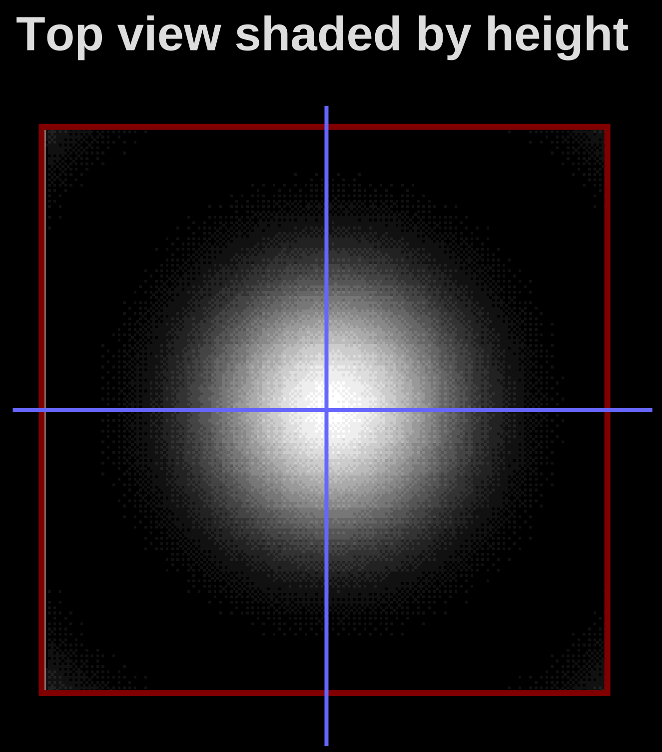
Radiometry Final Result

$$E_i = L_s \frac{\pi}{4}\left(\frac{d}{-f}\right)^2 \cos^4\alpha$$

- Image irradiance is proportional to:
  - Scene radiance L_s
  - Focal length of lens f
  - Diameter of lens d
    - f/d is often called the **f-number** of the lens
  - Off-axis angle α

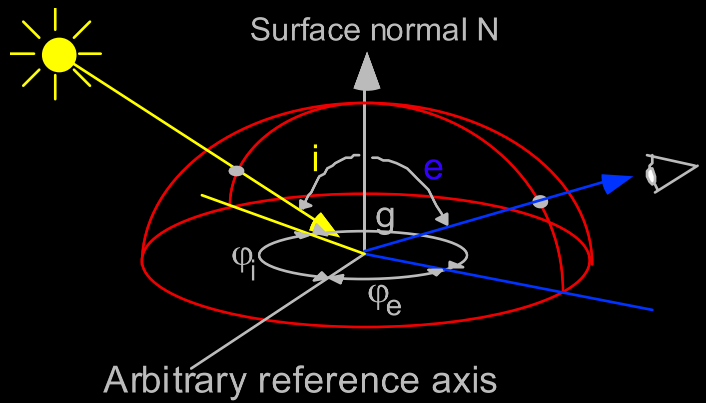
$\cos^4\alpha$ Light Falloff





Limitation of Radiometry Model

- Surface reflection ρ can be a function of viewing and/or illumination angle

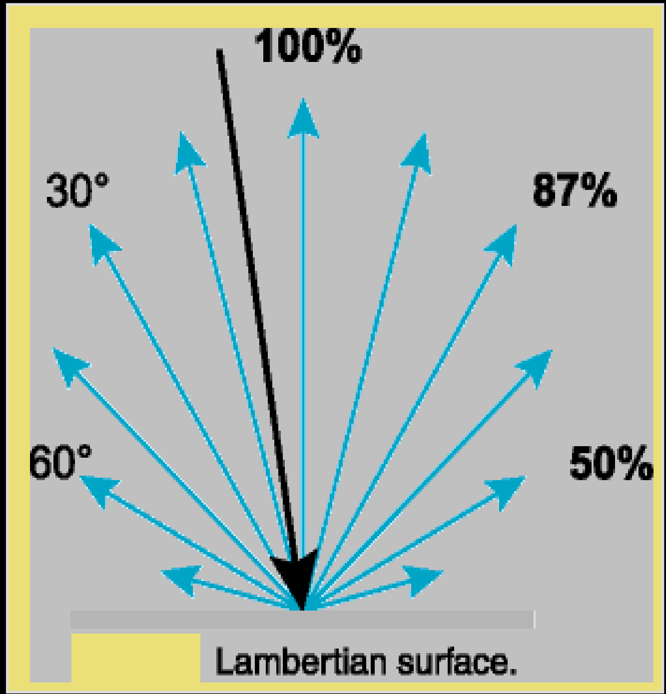


$$\rho(i,e,g,\varphi_e,\varphi_i) = \frac{dL(e,\varphi_e)}{dE(i,\varphi_i)}$$

- ρ may also be a function of the wavelength of the light source
- Assumed a point source (sky, for example, is not)

Lambertian Surfaces

- The BRDF for a Lambertian surface is a constant
  - $\rho(i,e,g,\varphi_e, \varphi_i) = k$
  - function of cos e due to the forshortening effect
  - k is the 'albedo' of the surface
  - Good model for diffuse surfaces
- Other models combine diffuse and specular components (Phong, Torrance-Sparrow, Oren-Nayar)
- References available upon request



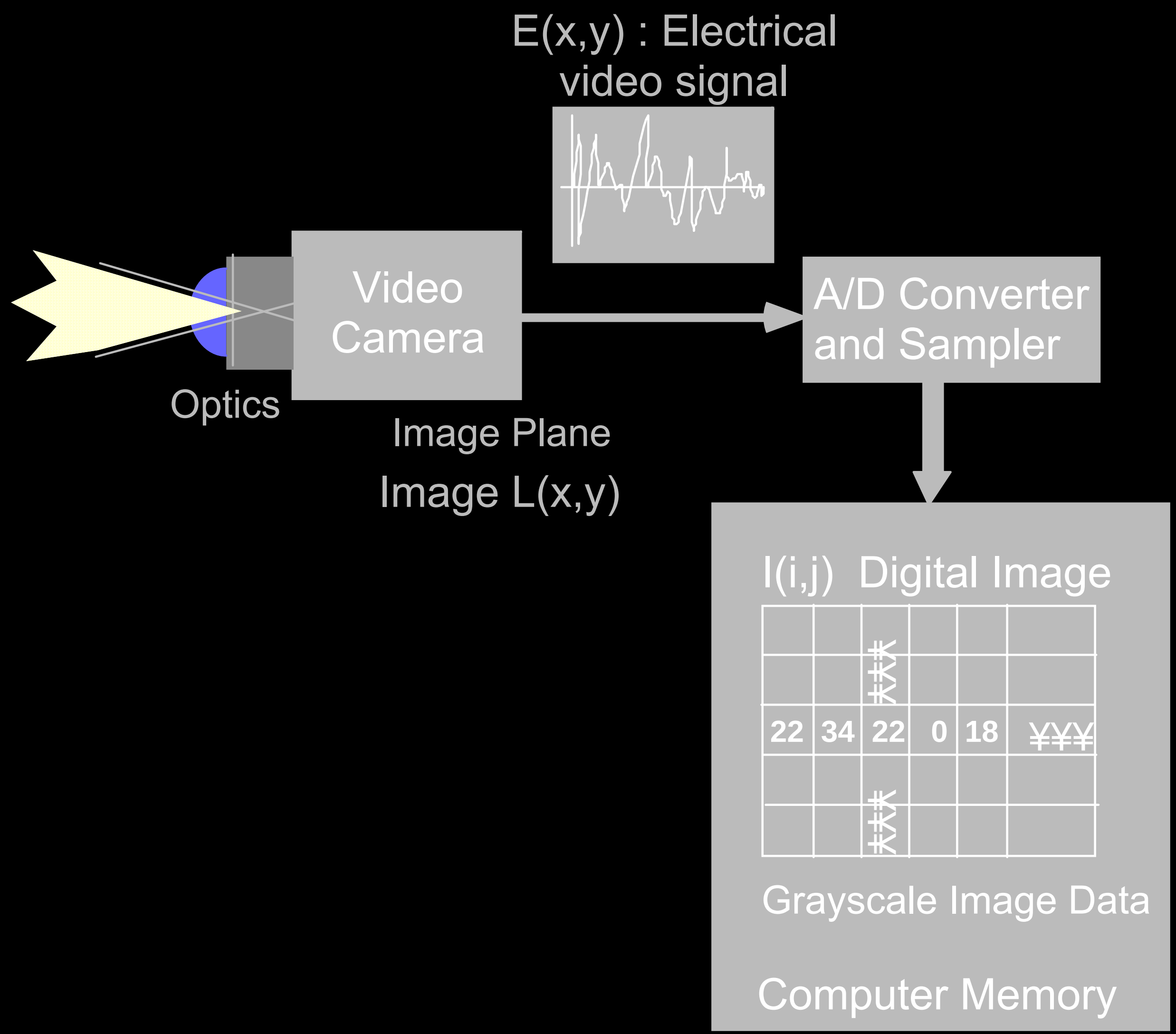
Photometry

- Photometry:

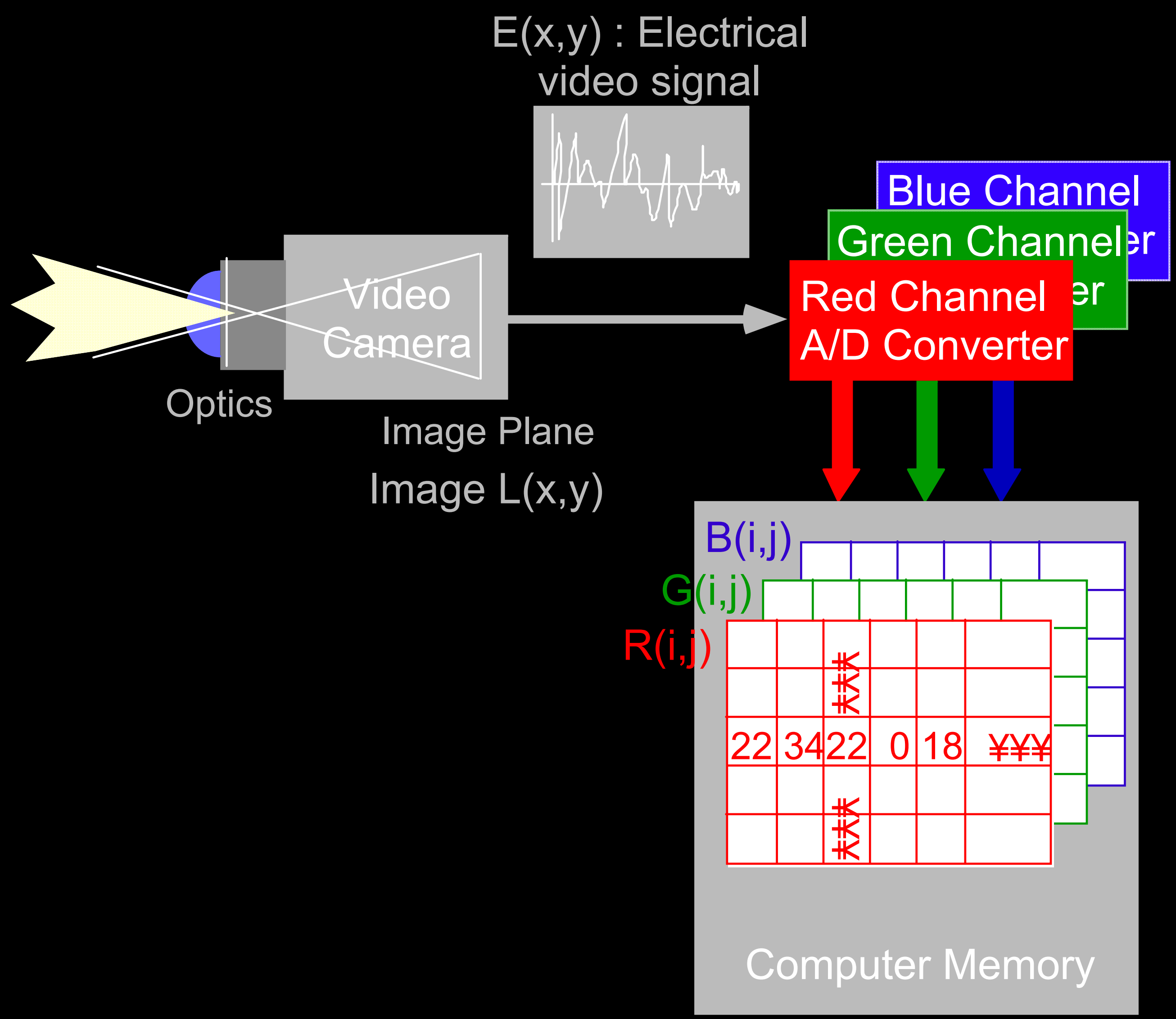
  Concerned with mechanisms for converting light energy into electrical energy.

World → Optics → Sensor →
Signal → Digitizer →
Digital Representation

B&W Video System

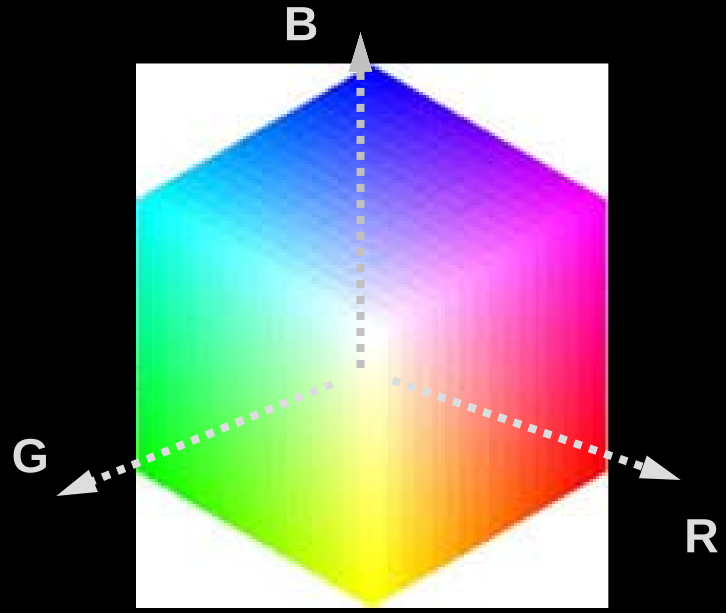


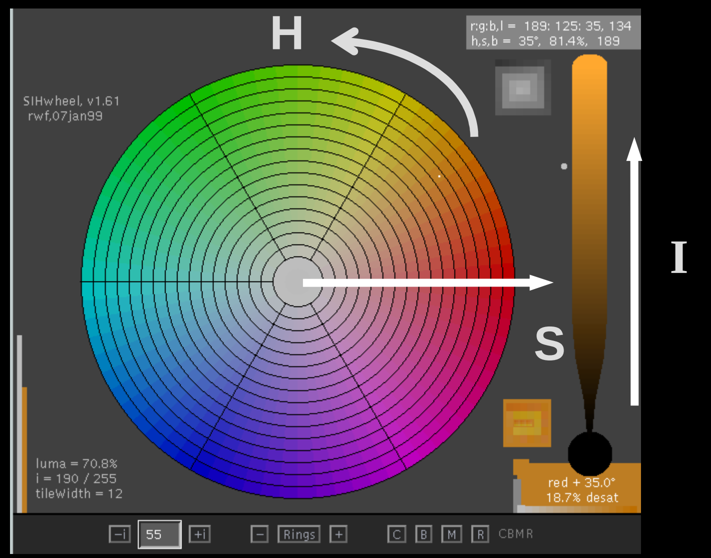
Color Video System



Color Representation

- Color Cube and Color Wheel





- For color spaces, please read
  - Color Cube http://www.morecrayons.com/palettes/webSmart/
  - Color Wheel http://home.att.net/~rocq/SIHwheel.html
  - http://www.netnam.vn/unescocourse/computervision/12.htm
  - http://www-viz.tamu.edu/faculty/parke/ends489f00/notes/sec1_4.html

Digital Color Cameras

- Three CCD-chips cameras
  - R, G, B separately, AND digital signals instead analog video
- One CCD Cameras
  - Bayer color filter array
  - http://www.siliconimaging.com/RGB%20Bayer.htm
  - http://www.fillfactory.com/htm/technology/htm/rgbfaq.htm
- Image Format with Matlab (show demo)

Spectral Sensitivity

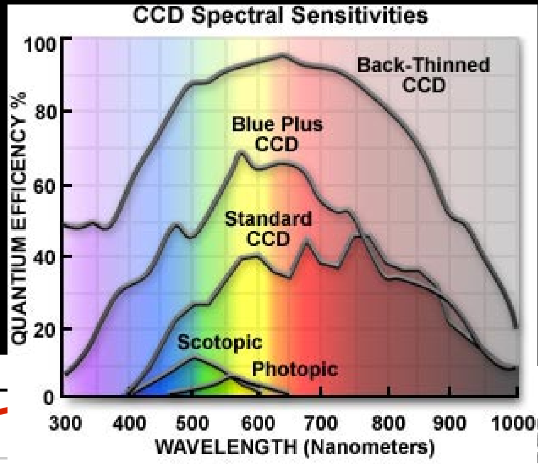
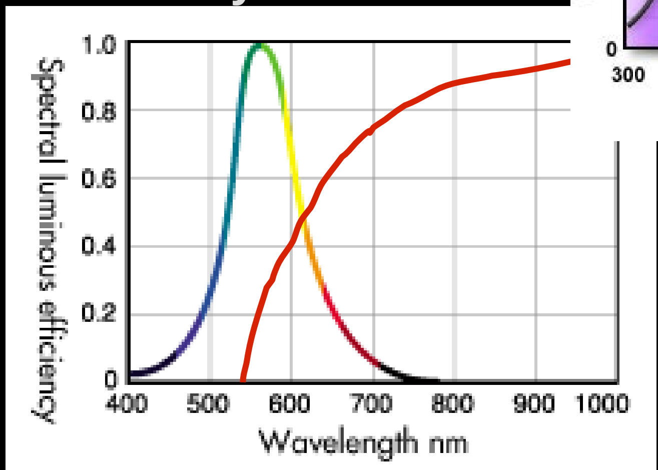
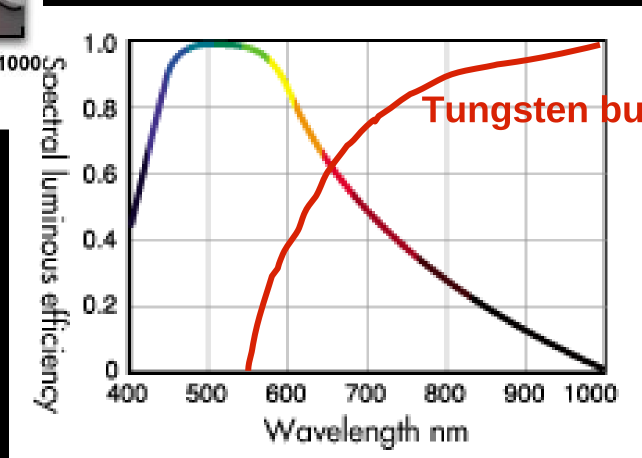


Figure 1

Human Eye

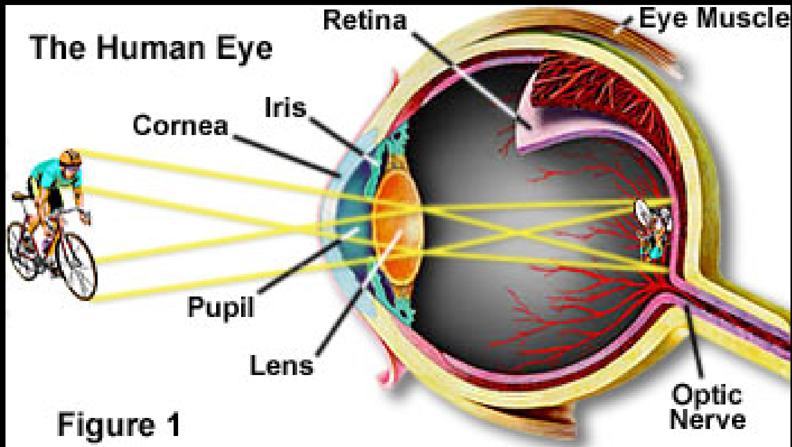


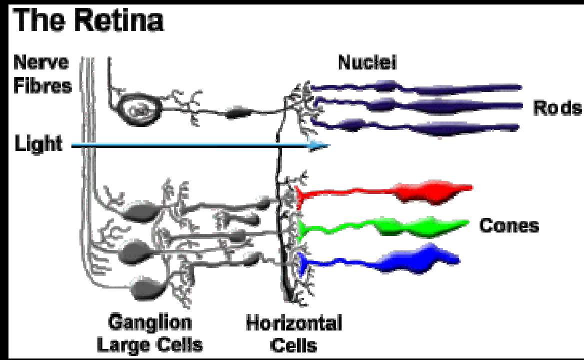
CCD Camera

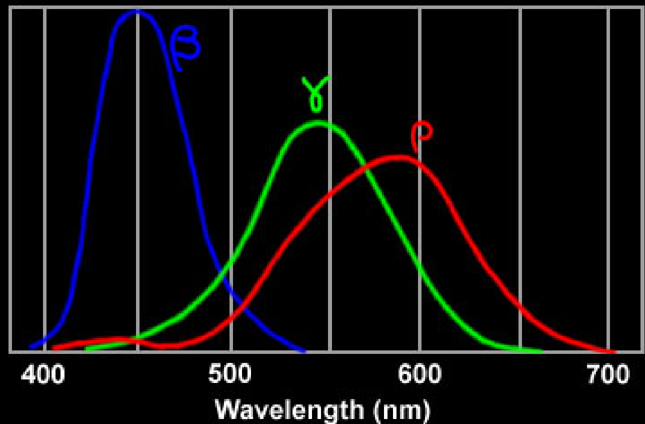


- Figure 1 shows relative efficiency of conversion for the eye (scotopic and photopic curves) and several types of CCD cameras. Note the CCD cameras are much more sensitive than the eye.
- Note the enhanced sensitivity of the CCD in the Infrared and Ultraviolet (bottom two figures)
- Both figures also show a hand-drawn sketch of the spectrum of a tungsten light bulb

Human Eyes and Color Perception







- Visit a cool site with Interactive Java tutorial:
  - http://micro.magnet.fsu.edu/primer/lightandcolor/vision.html
- Another site about human color perception:
  - http://www.photo.net/photo/edscott/vis00010.htm

Characteristics

- In general, $V(x,y) = k\,E(x,y)^{\gamma}$ where
  - k is a constant
  - **γ is a parameter of the type of sensor**
    - **$\gamma=1$ (approximately) for a CCD camera**
    - **$\gamma=.65$ for an old type vidicon camera**
- Factors influencing performance:
  - Optical distortion: pincushion, barrel, non-linearities
  - Sensor dynamic range (30:1 CCD, 200:1 vidicon)
  - Sensor Shading (nonuniform responses from different locations)
- **TV Camera pros: cheap, portable, small size**
- **TV Camera cons: poor signal to noise, limited dynamic range, fixed array size with small image (getting better)**

Sensor Performance

- Optical Distortion: pincushion, barrel, non-linearities
- Sensor Dynamic Range: (30:1 for a CCD, 200:1 Vidicon)
- Sensor Blooming: spot size proportional to input intensity
- Sensor Shading: (non-uniform response at outer edges of image)
- Dead CCD cells

There is no "universal sensor". Sensors must be selected/tuned for a particular domain and application.

Lens Aberrations

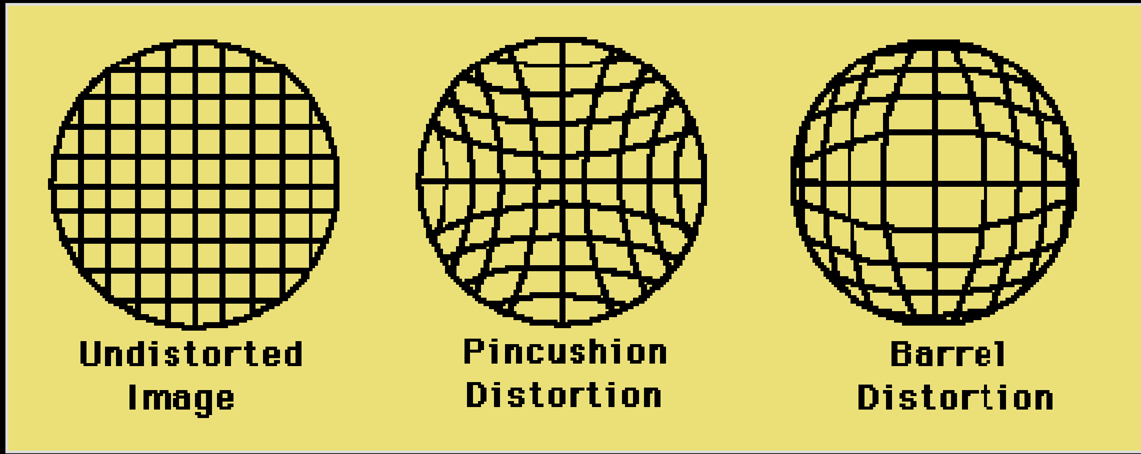
- In an ideal optical system, all rays of light from a point in the object plane would converge to the same point in the image plane, forming a clear image.
- The lens defects which cause different rays to converge to different points are called aberrations.

  - Distortion: barrel, pincushion
  - Curvature of field
  - Chromatic Aberration
  - Spherical aberration
  - Coma
  - Astigmatism

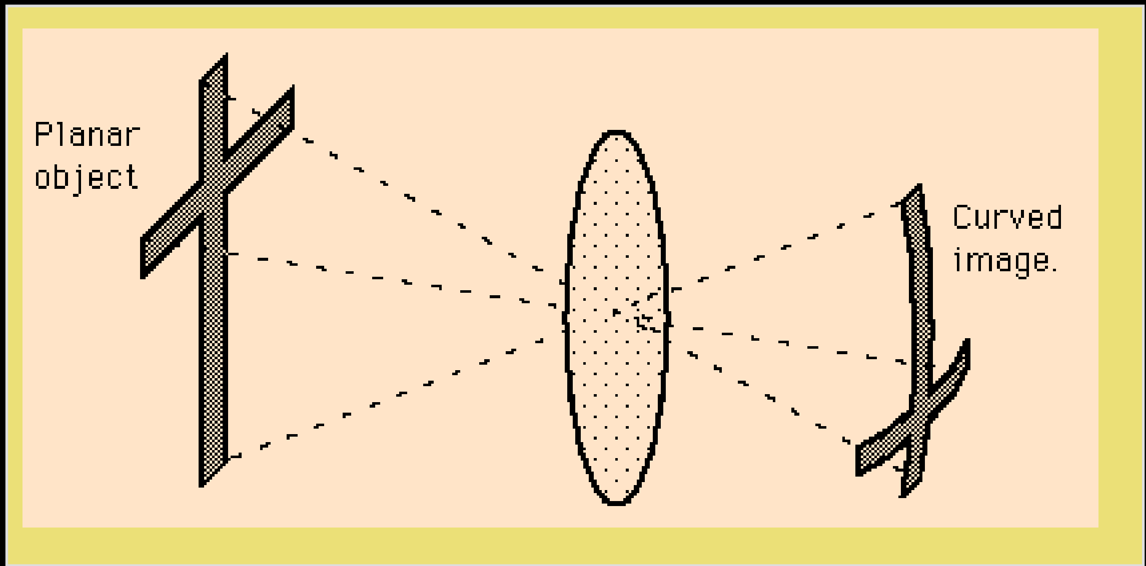
Aberration slides after http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/aberrcon.html#c1

Lens Aberrations

- Distortion

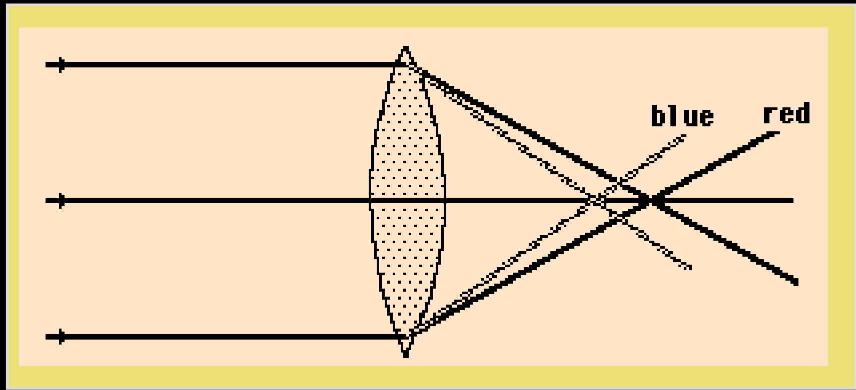


- Curved Field



Lens Aberrations

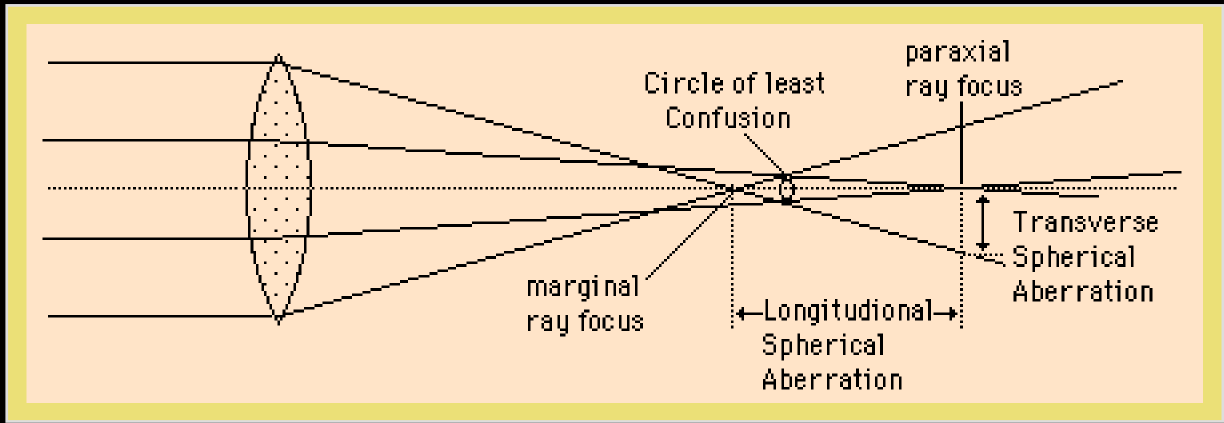
- Chromatic Aberration



- Focal Length of lens depends on refraction and
- The index of refraction for blue light (short wavelengths) is larger than that of red light (long wavelengths).
- Therefore, a lens will not focus different colors in exactly the same place
- The amount of chromatic aberration depends on the dispersion (change of index of refraction with wavelength) of the glass.

Lens Aberration

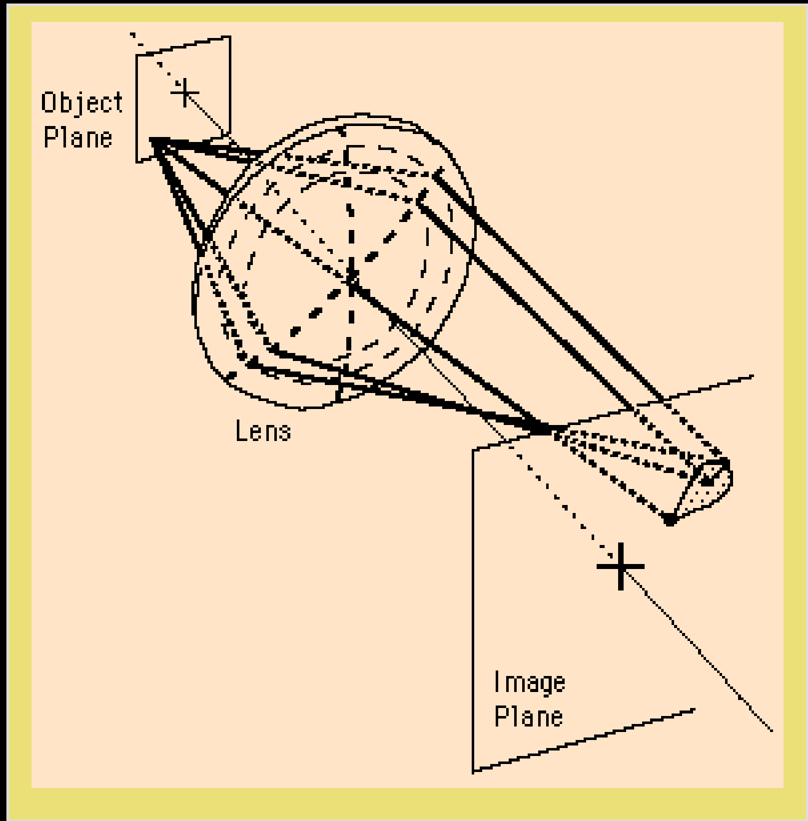
- Spherical Aberration



- Rays which are parallel to the optic axis but at different distances from the optic axis fail to converge to the same point.

Lens Aberrations

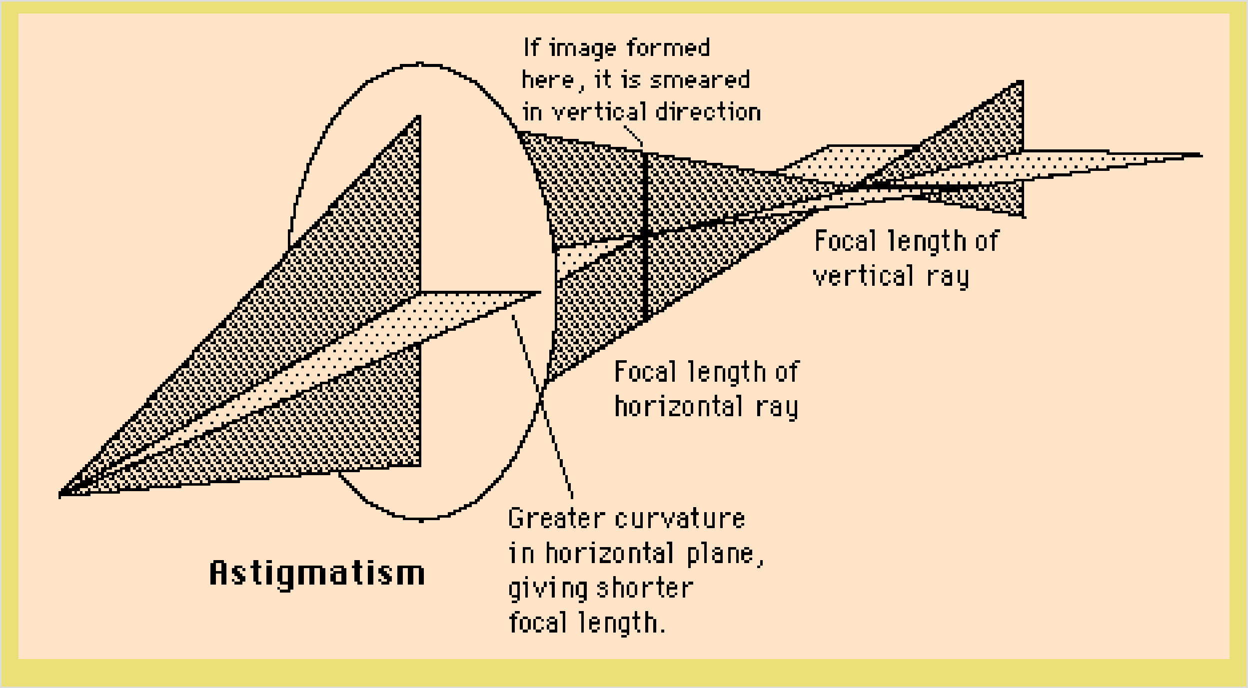
- Coma



- Rays from an off-axis point of light in the object plane create a trailing "comet-like" blur directed away from the optic axis
- Becomes worse the further away from the central axis the point is

Lens Aberrations

- Astigmatism



- Results from different lens curvatures in different planes.

Sensor Summary

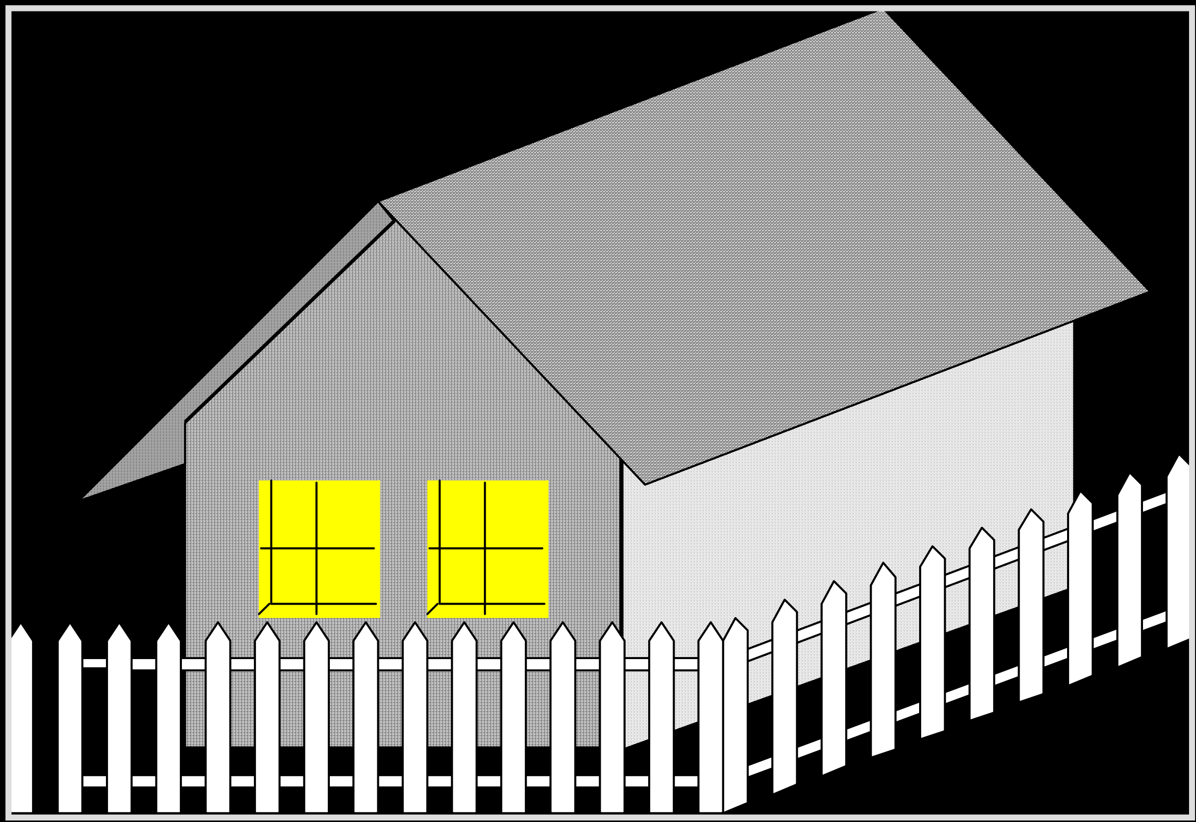
- Visible Light/Heat
  - Camera/Film combination
  - Digital Camera
  - Video Cameras
  - FLIR (Forward Looking Infrared)
- Range Sensors
  - Radar (active sensing)
    - sonar
    - laser
  - Triangulation
    - stereo
    - structured light
      - – striped, patterned
    - Moire
    - Holographic Interferometry
    - Lens Focus
    - Fresnel Diffraction
- Others
- Almost anything which produces a 2d signal that is related to the scene can be used as a sensor

Digitization

World ⇒ Optics ⇒ Sensor ⇒
Signal ⇒ Digitizer ⇒
Digital Representation

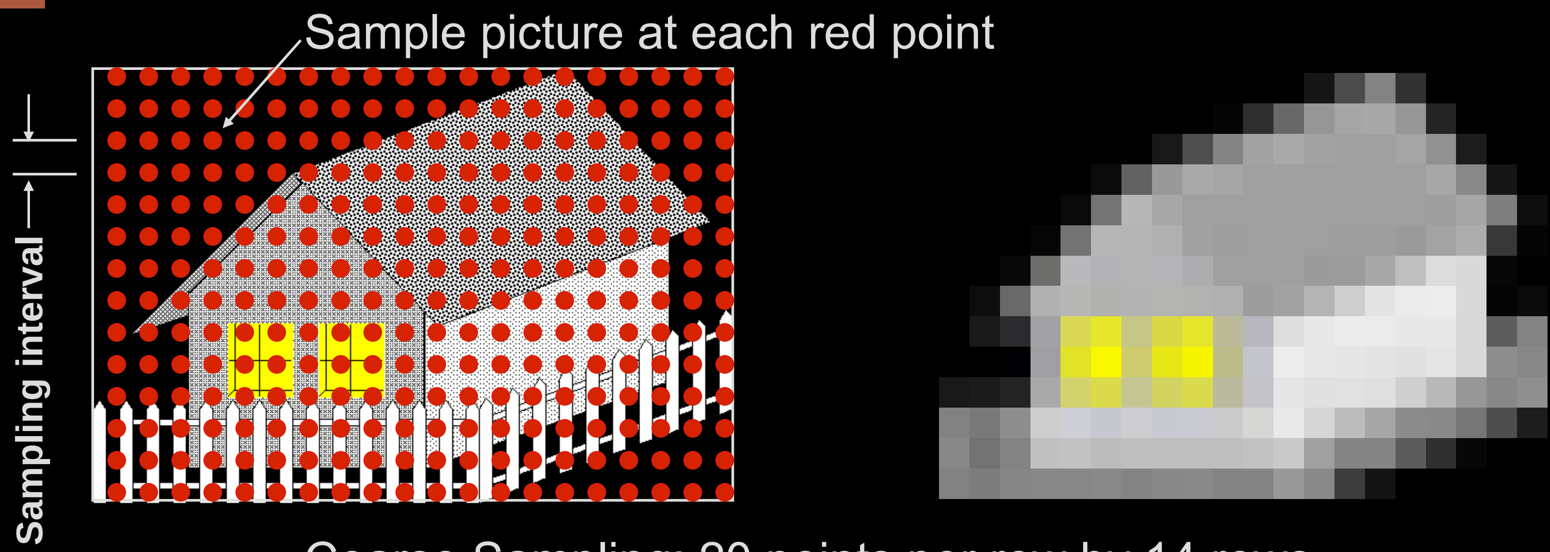
- Digitization: conversion of the continuous (in space and value) electrical signal into a digital signal (digital image)
- Three decisions must be made:
  - Spatial resolution (how many samples to take)
  - Signal resolution (dynamic range of values- quantization)
  - Tessellation pattern (how to 'cover' the image with sample points)

Digitization: Spatial Resolution

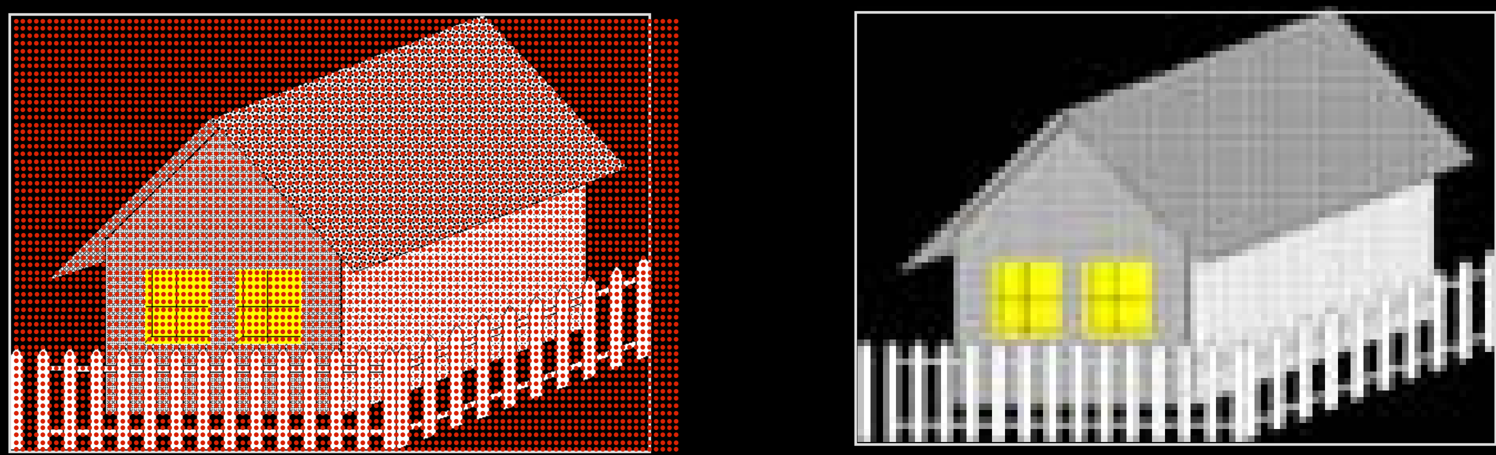


- Let's digitize this image
  - Assume a square sampling pattern
  - Vary density of sampling grid

Spatial Resolution



Coarse Sampling: 20 points per row by 14 rows

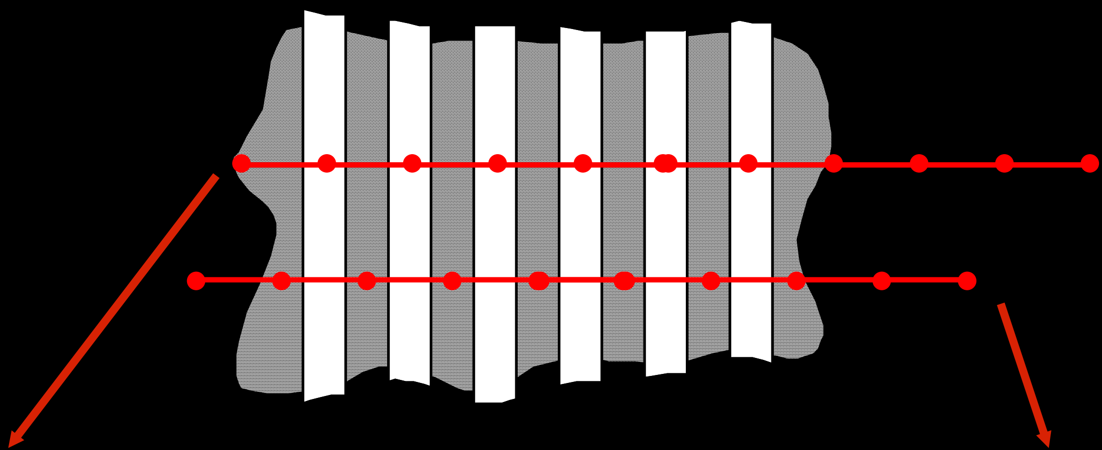


Finer Sampling: 100 points per row by 68 rows

Effect of Sampling Interval - 1

- Look in vicinity of the picket fence:

Sampling Interval:



100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

White Image!

NO EVIDENCE
OF THE FENCE!

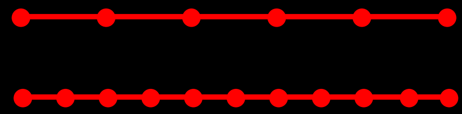
40	40	40	40	40	40
40	40	40	40	40	40
40	40	40	40	40	40
40	40	40	40	40	40

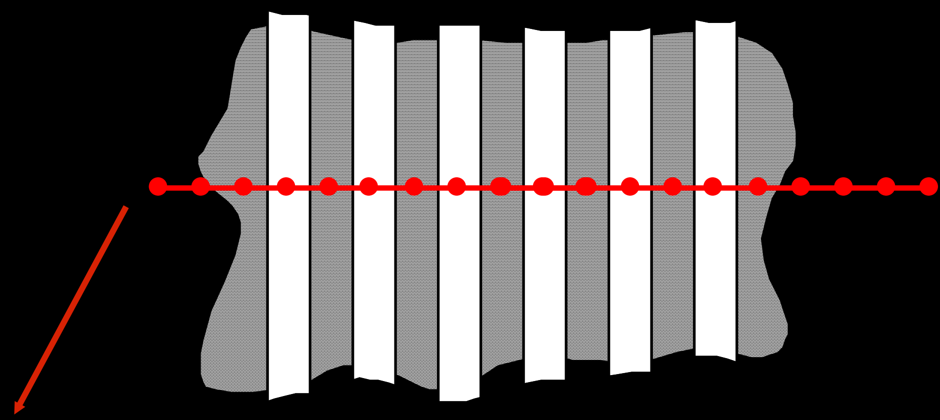
Dark Gray Image!

Effect of Sampling Interval - 2

- Look in vicinity of picket fence:

Sampling Interval:





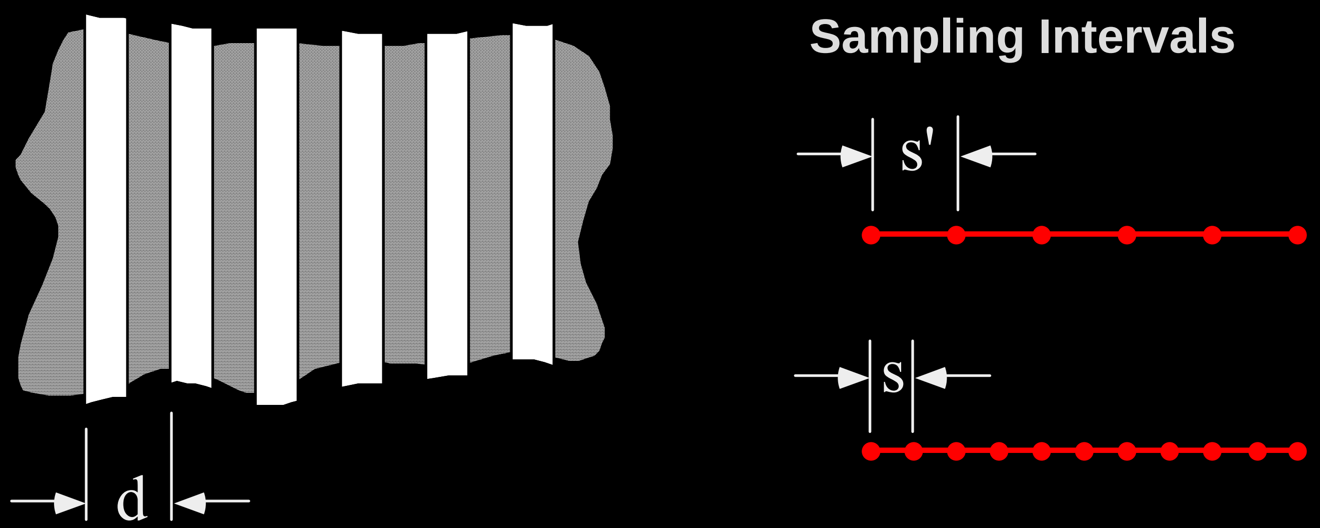
40	100	40	100	40
40	100	40	100	40
40	100	40	100	40
40	100	40	100	40

Now we've got a fence!

What's the difference between this attempt and the last one?

The Missing Fence Found

- Consider the repetitive structure of the fence:



Case 1: s' = d The sampling interval is equal to the size of the repetitive structure **NO FENCE**

Case 2: s = d/2 The sampling interval is one-half the size of the repetitive structure **FENCE**

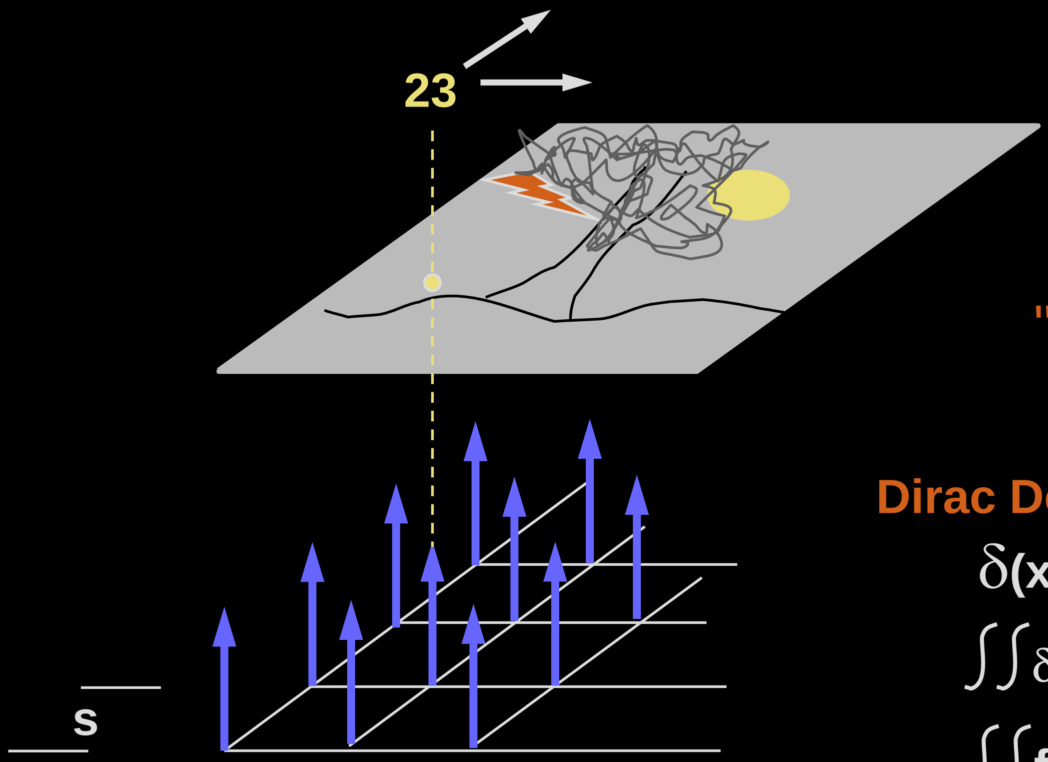
The Sampling Theorem

- IF: the size of the smallest structure to be preserved is d
- THEN: the sampling interval must be smaller than d/2

- Can be shown to be true mathematically
- Repetitive structure has a certain frequency ('pickets/foot')
  - To preserve structure must sample at twice the frequency
  - Holds for images, audio CDs, digital television….
- Leads naturally to Fourier Analysis (later in course)

Sampling

- Rough Idea: Ideal Case



"Digitized Image"

"Continuous Image"

Dirac Delta Function 2D "Comb"

$\delta(x,y) = 0$ for $x \neq 0, y \neq 0$

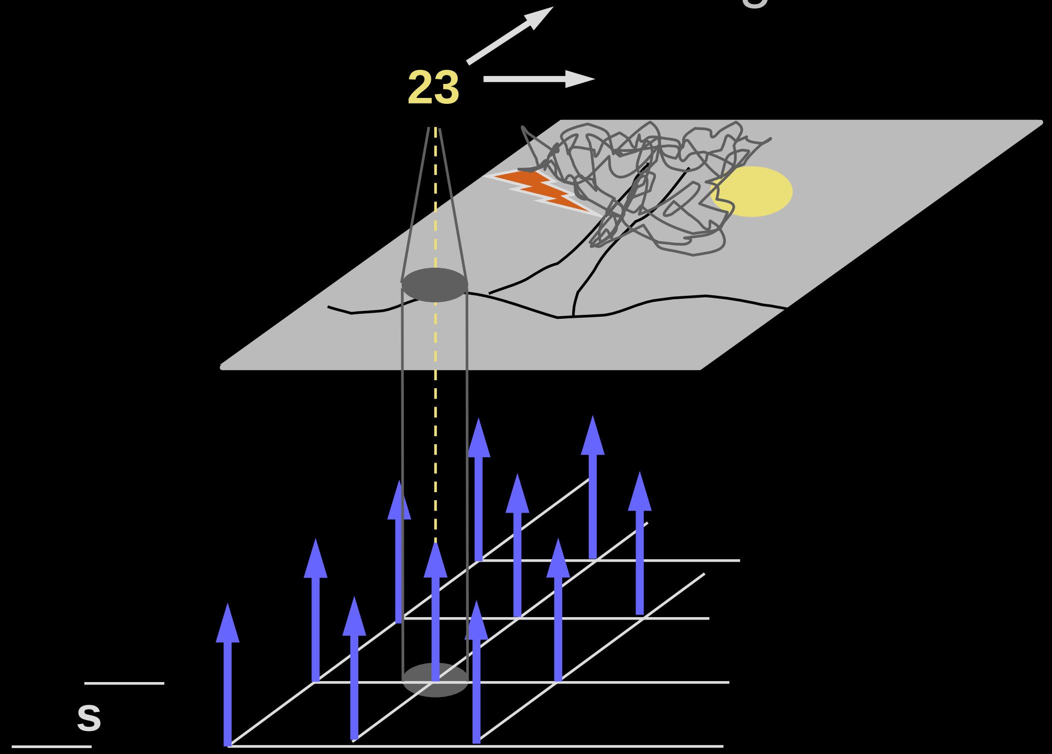
$\int\int \delta(x,y)\, dx\, dy = 1$

$\int\int f(x,y)\delta(x-a,y-b)\, dx\, dy = f(a,b)$

$\delta(x-ns,y-ns)$ for $n = 1\ldots.32$ (e.g.)

Sampling

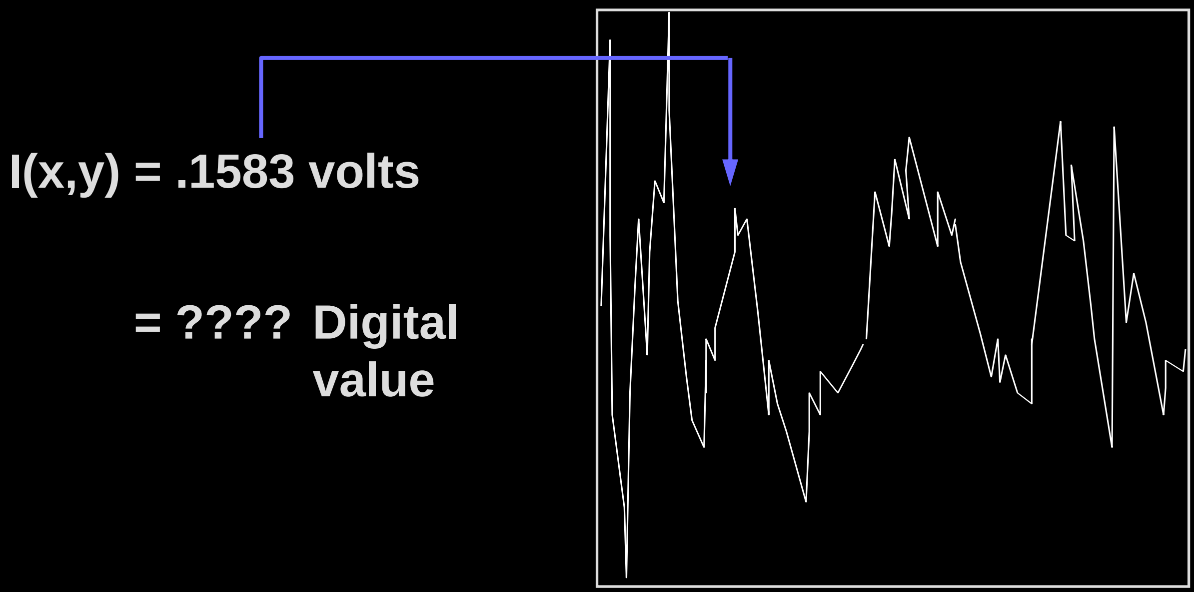
- Rough Idea: Actual Case
  - Can't realize an ideal point function in real equipment
  - "Delta function" equivalent has an area
  - Value returned is the average over this area



Mixed Pixel Problem

Signal Quantization

- Goal: determine a mapping from a continuous signal (e.g. analog video signal) to one of K discrete (digital) levels.



Quantization

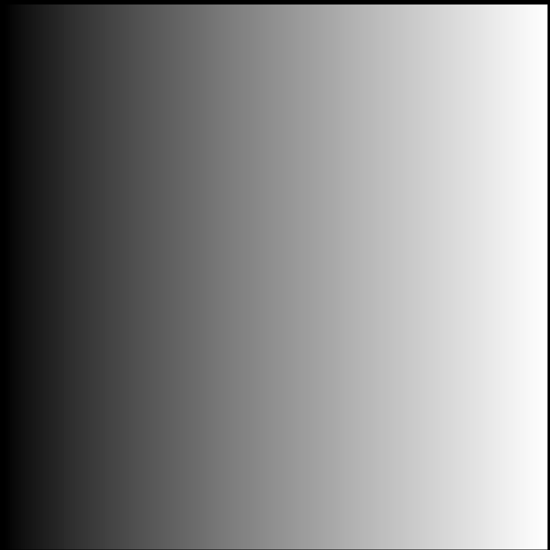
- $I(x,y)$ = continuous signal: $0 \leq I \leq M$
- Want to quantize to K values 0,1,....K-1
- K usually chosen to be a power of 2:

K	#Levels	#Bits
2	2	1
4	4	2
8	8	3
16	16	4
32	32	5
64	64	6
128	128	7
256	256	8

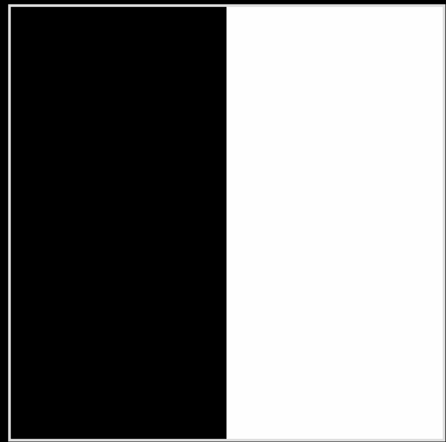
- Mapping from input signal to output signal is to be determined.
- Several types of mappings: uniform, logarithmic, etc.

Choice of K

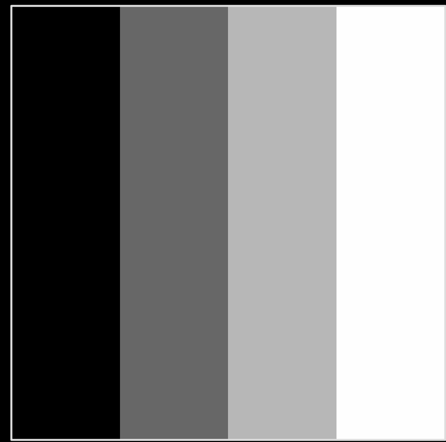
Original



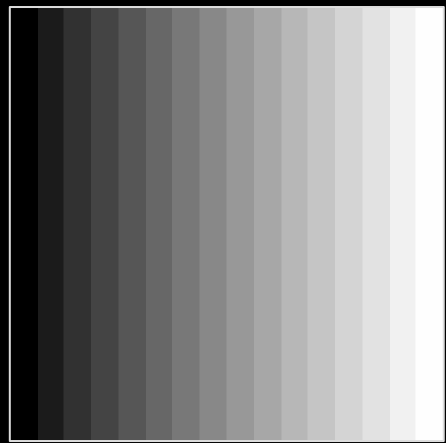
Linear Ramp



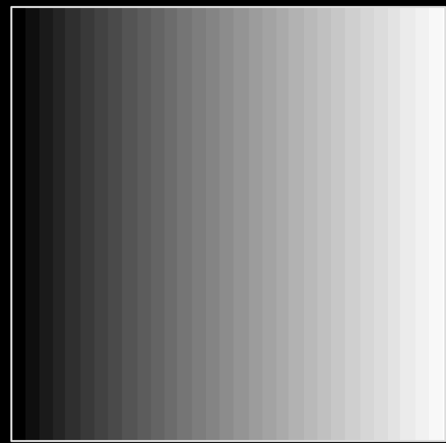
K=2



K=4



K=16



K=32

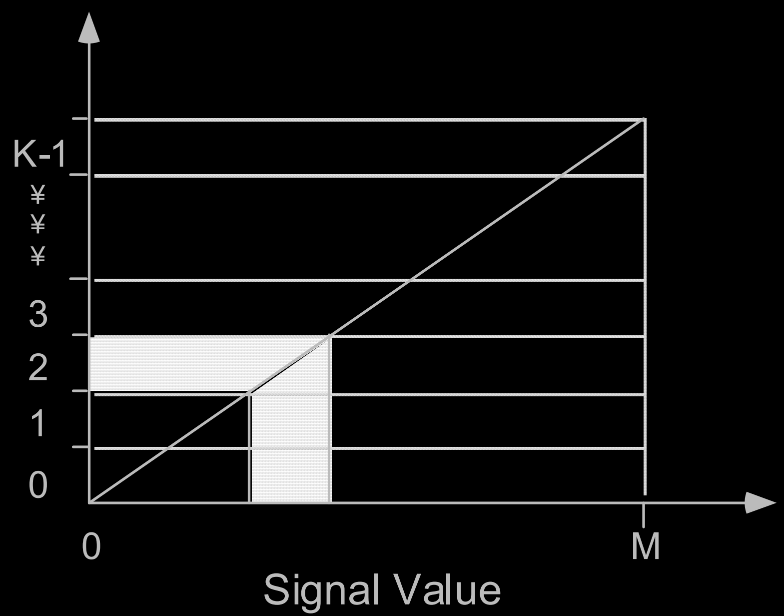
Choice of K

K=2 (each color)

K=4 (each color)

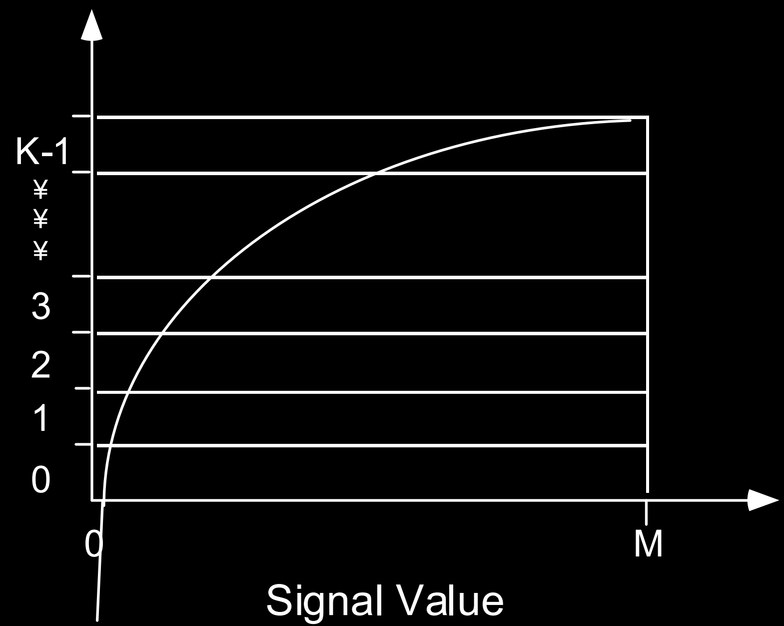
Choice of Function: Uniform

- Uniform sampling divides the signal range [0-M] into K equal-sized intervals.
- The integers 0,...K-1 are assigned to these intervals.
- All signal values within an interval are represented by the associated integer value.
- Defines a mapping:



Logarithmic Quantization

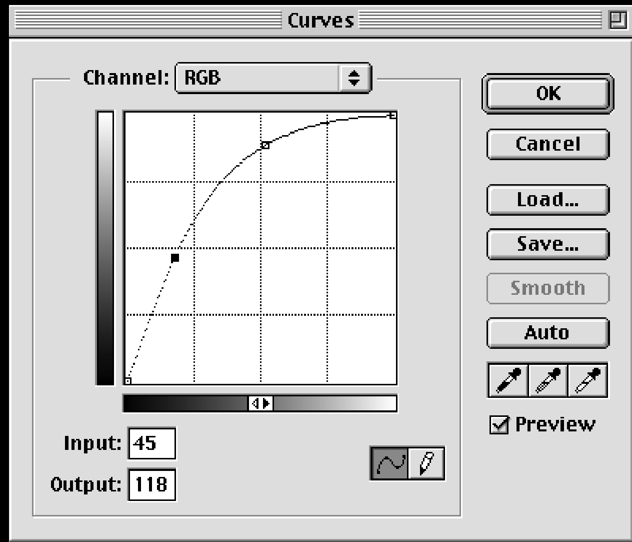
- Signal is log I(x,y).
- Effect is:



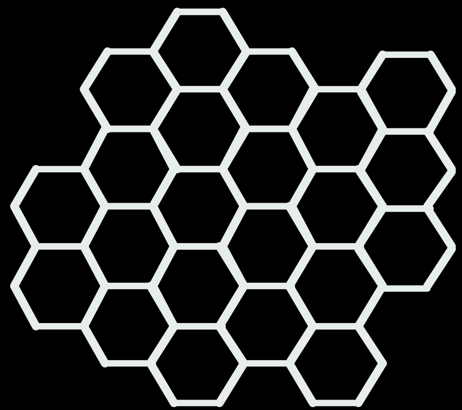
- Detail enhanced in the low signal values at expense of detail in high signal values.

Logarithmic Quantization

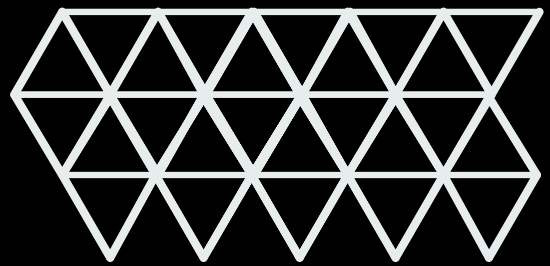
Quantization Curve



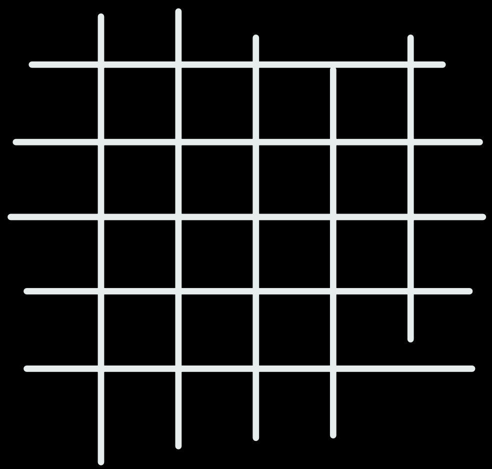
Tesselation Patterns



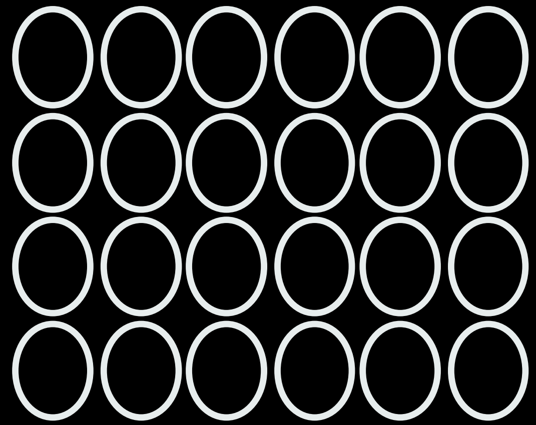
Hexagonal



Triangular

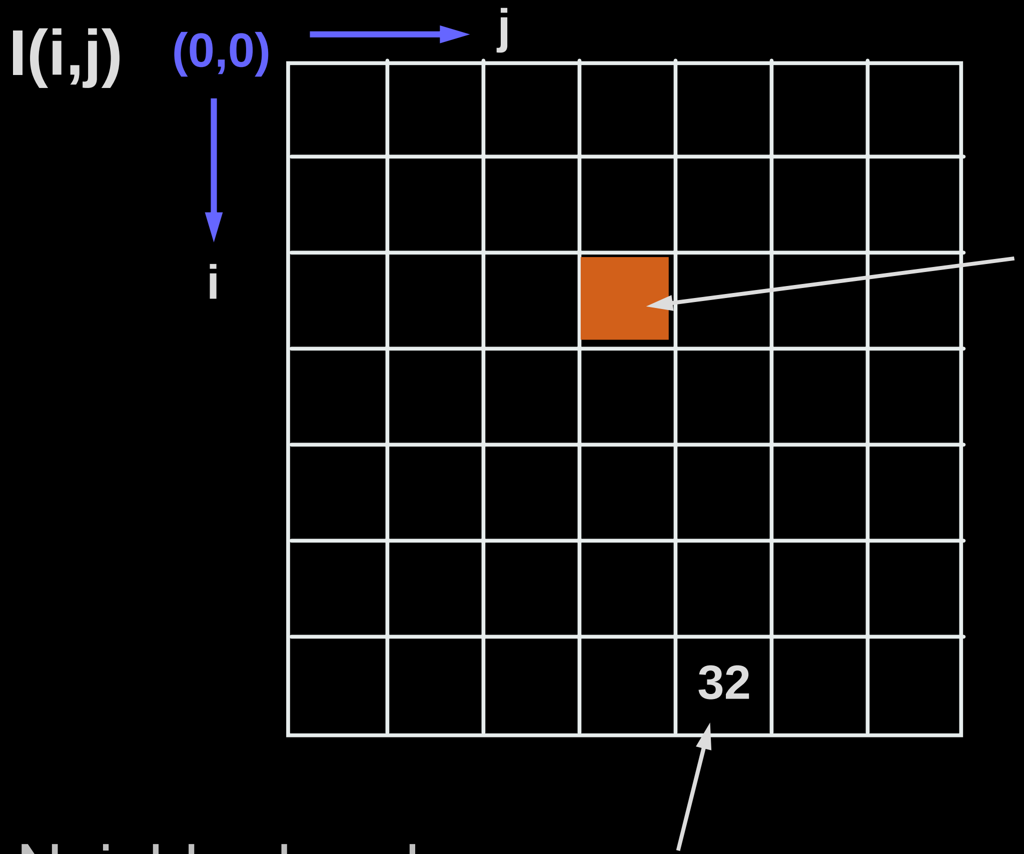


Rectangular



Typical

Digital Geometry



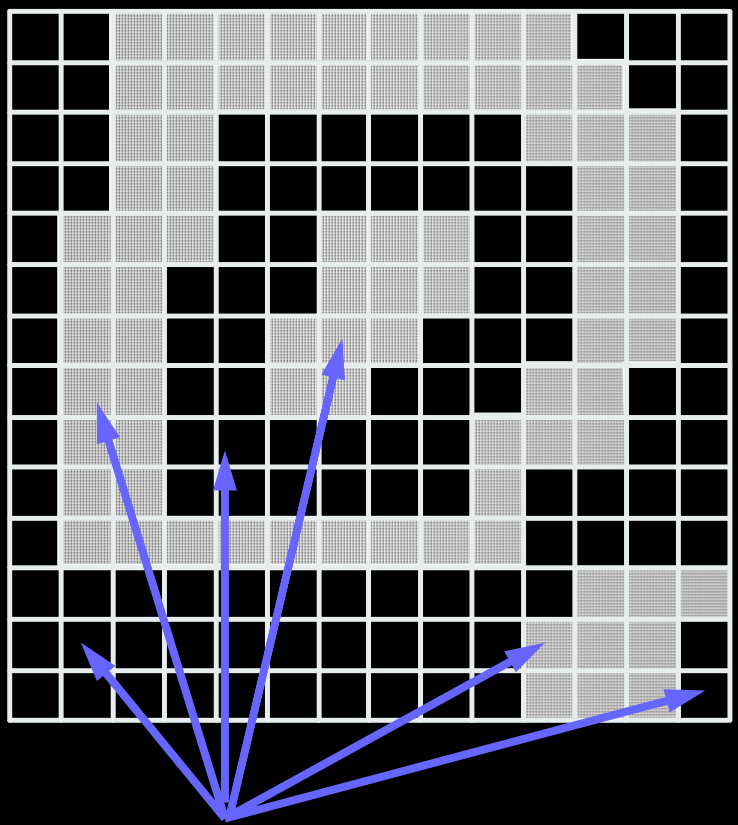
Picture Element or Pixel

Pixel value I(I,j) =
- 0,1 Binary Image
- 0 - K-1 Gray Scale Image
- Vector: Multispectral Image

- Neighborhood
- Connectedness
- Distance Metrics

Connected Components

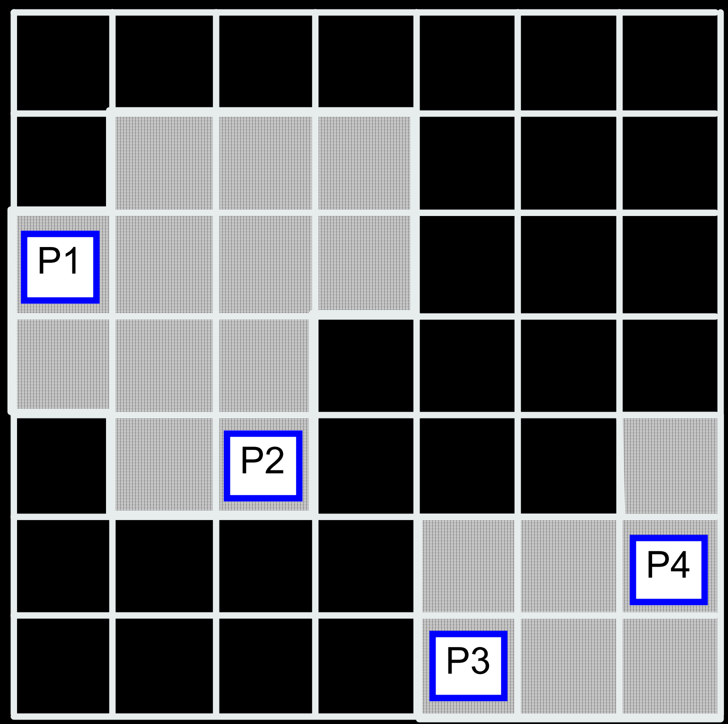
- Binary image with multiple 'objects'
- Separate 'objects' must be labeled individually



6 Connected Components

Finding Connected Components

- Two points in an image are 'connected' if a path can be found for which the value of the image function is the same all along the path.



P_1 connected to P_2

P_3 connected to P_4

P_1 not connected to P_3 or P_4

P_2 not connected to P_3 or P_4

P_3 not connected to P_1 or P_2

P_4 not connected to P_1 or P_2

Algorithm

- Pick any pixel in the image and assign it a label
- Assign same label to any neighbor pixel with the same value of the image function
- Continue labeling neighbors until no neighbors can be assigned this label
- Choose another label and another pixel not already labeled and continue
- If no more unlabeled image points, stop.

Who's my neighbor?

Example

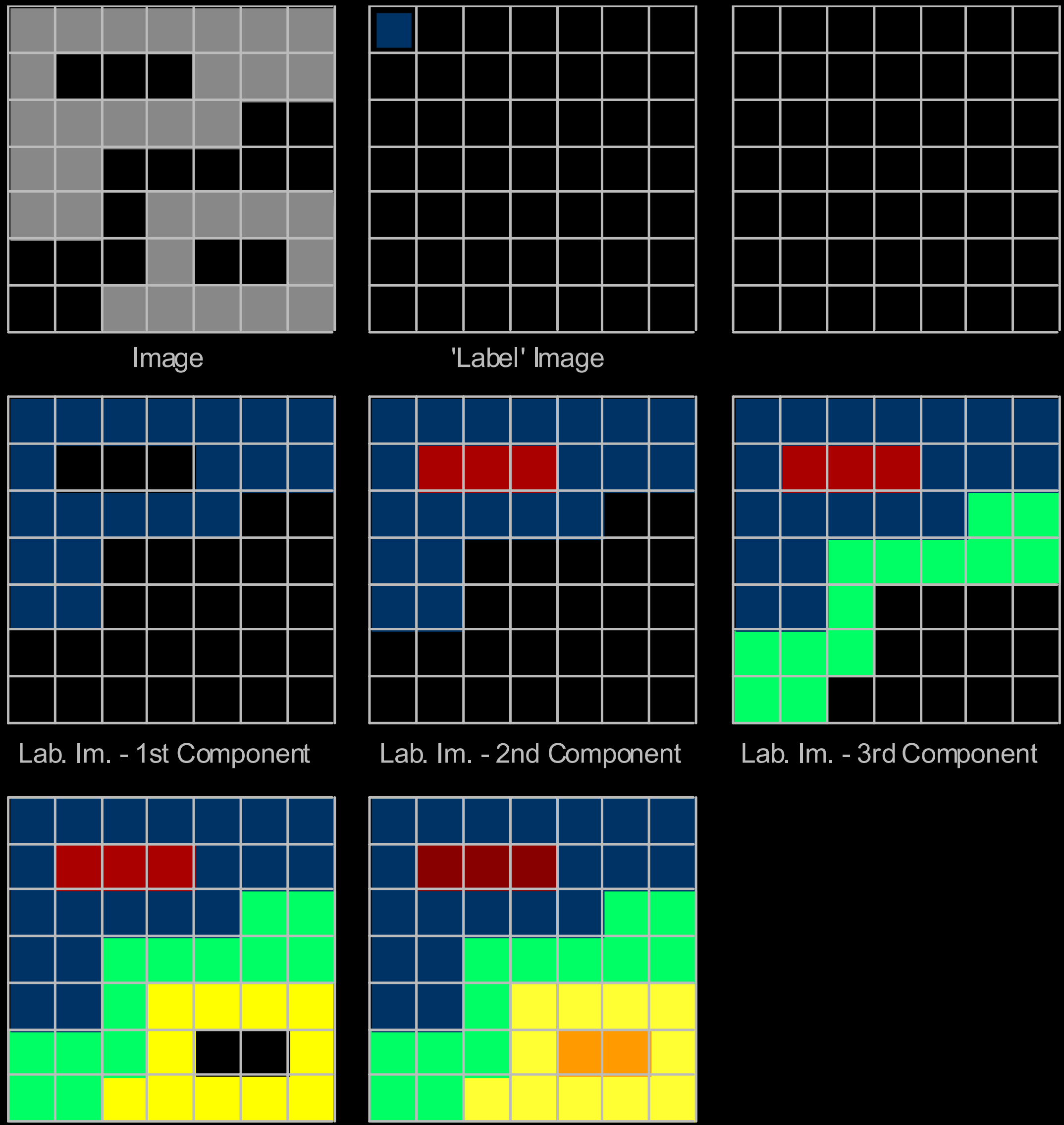


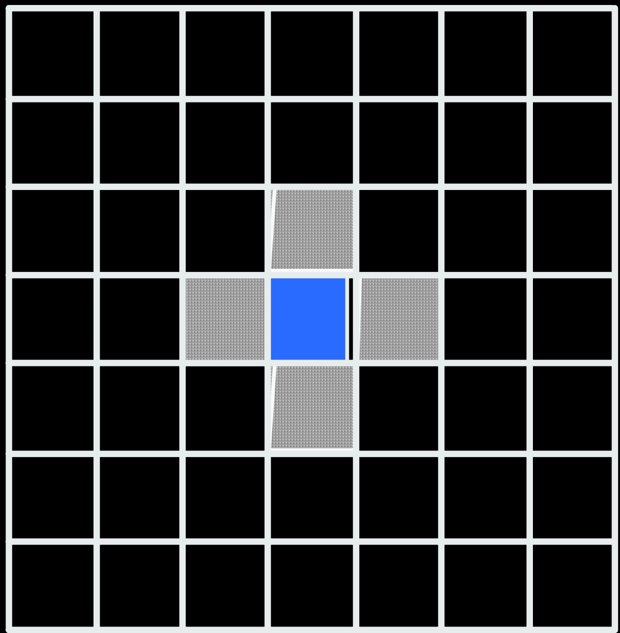
Image — 'Label' Image

Lab. Im. - 1st Component — Lab. Im. - 2nd Component — Lab. Im. - 3rd Component

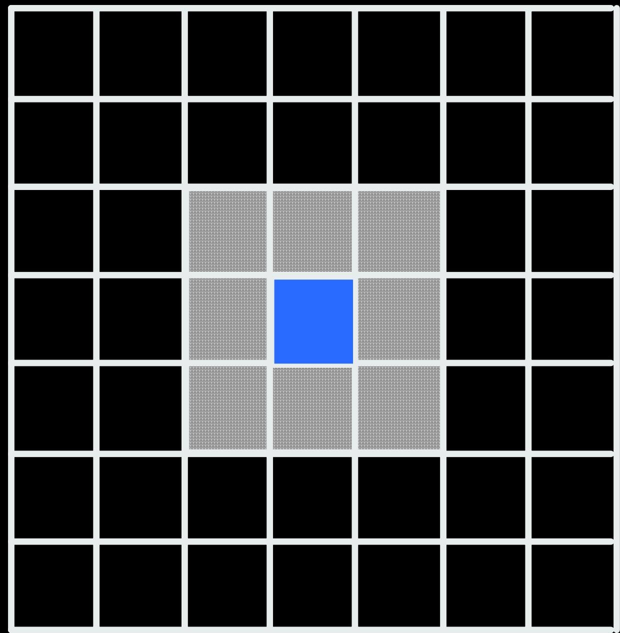
Lab. Im. - 4th Component — Final Labeling

Neighbor

- Consider the definition of the term 'neighbor'
- Two common definitions:



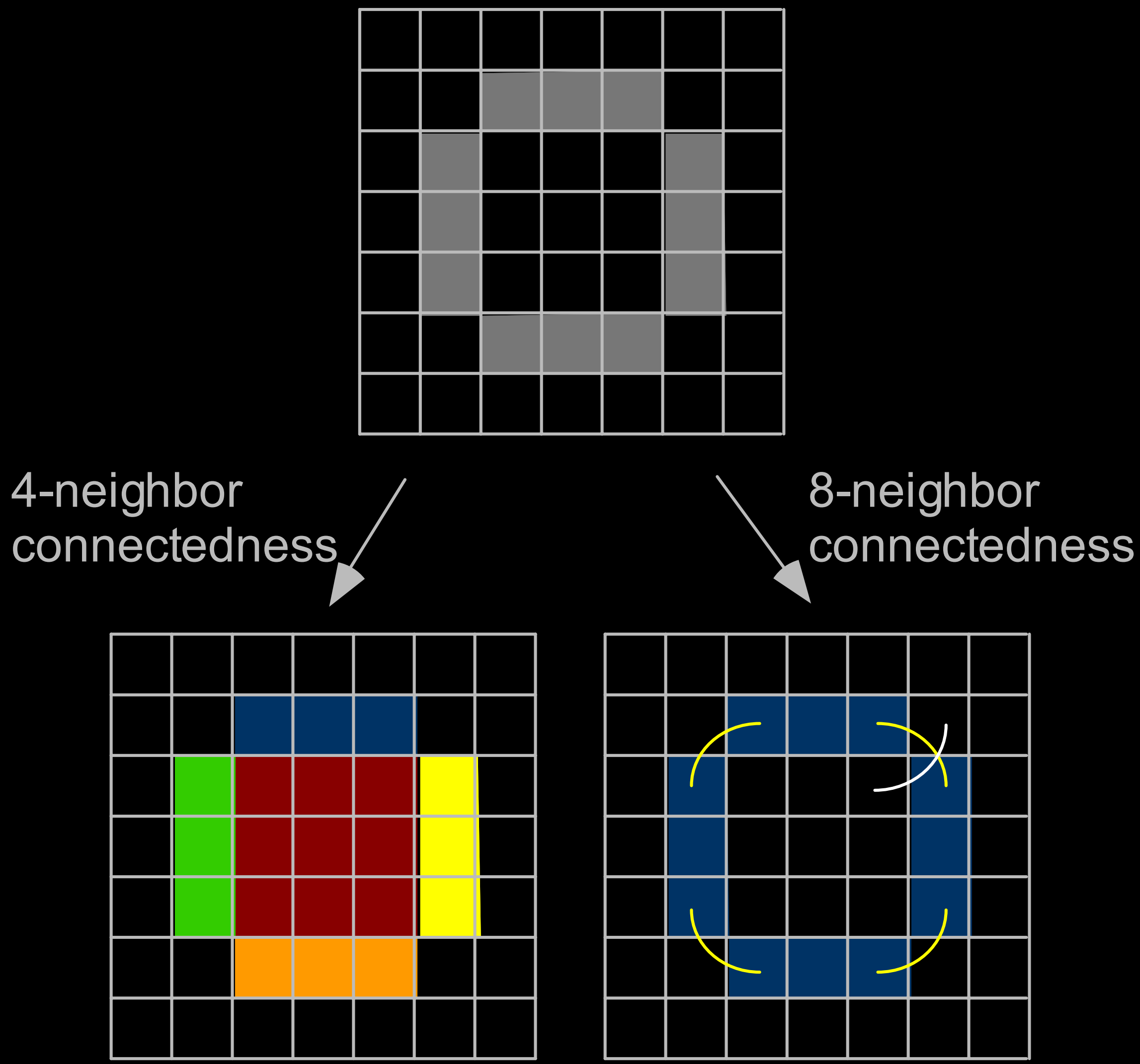
Four Neighbor



Eight Neighbor

- Consider what happens with a closed curve.
- One would expect a closed curve to partition the plane into two connected regions.

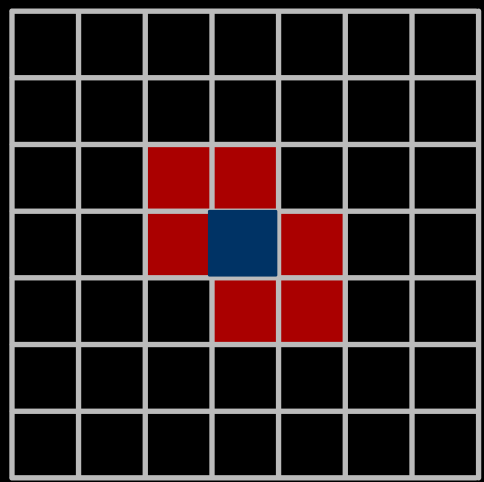
Alternate Neighborhood Definitions



Neither neighborhood definition satisfactory!

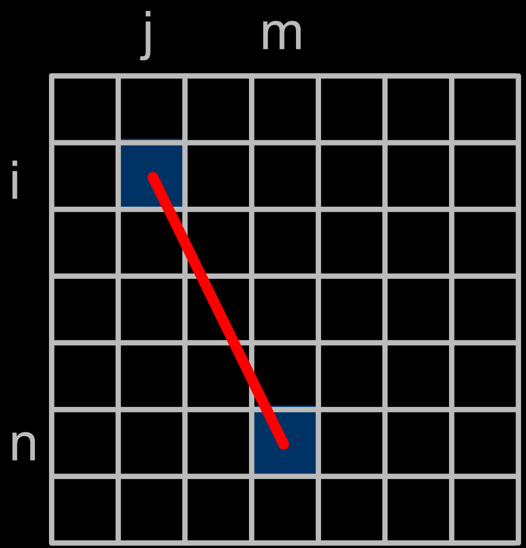
Possible Solutions

- Use 4-neighborhood for object and 8-neighborhood for background
  - requires a-priori knowledge about which pixels are object and which are background
- Use a six-connected neighborhood:



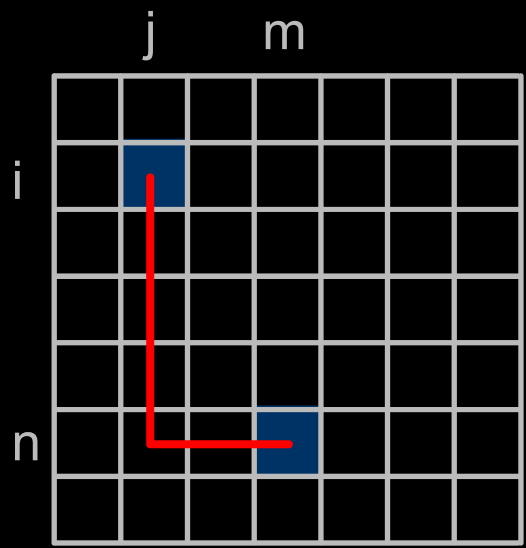
Digital Distances

- Alternate distance metrics for digital images



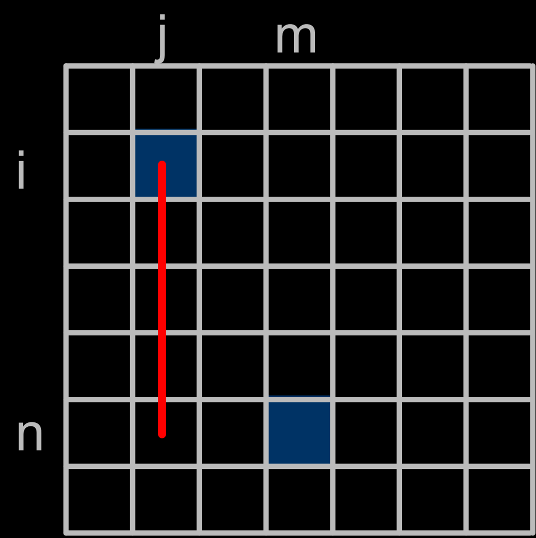
Euclidean Distance

$$= \sqrt{(i-n)^2 + (j-m)^2}$$



City Block Distance

$$= |i-n| + |j-m|$$



Chessboard Distance

$$= \max[\ |i-n|, |j-m|\ ]$$

Next:
Features