Machine Vision

lecture 5. Part 4

Filtering in the Frequency Domain

Based on lectures of

Brian Mac Namee

Contents

In this lecture we will look at image enhancement in the frequency domain

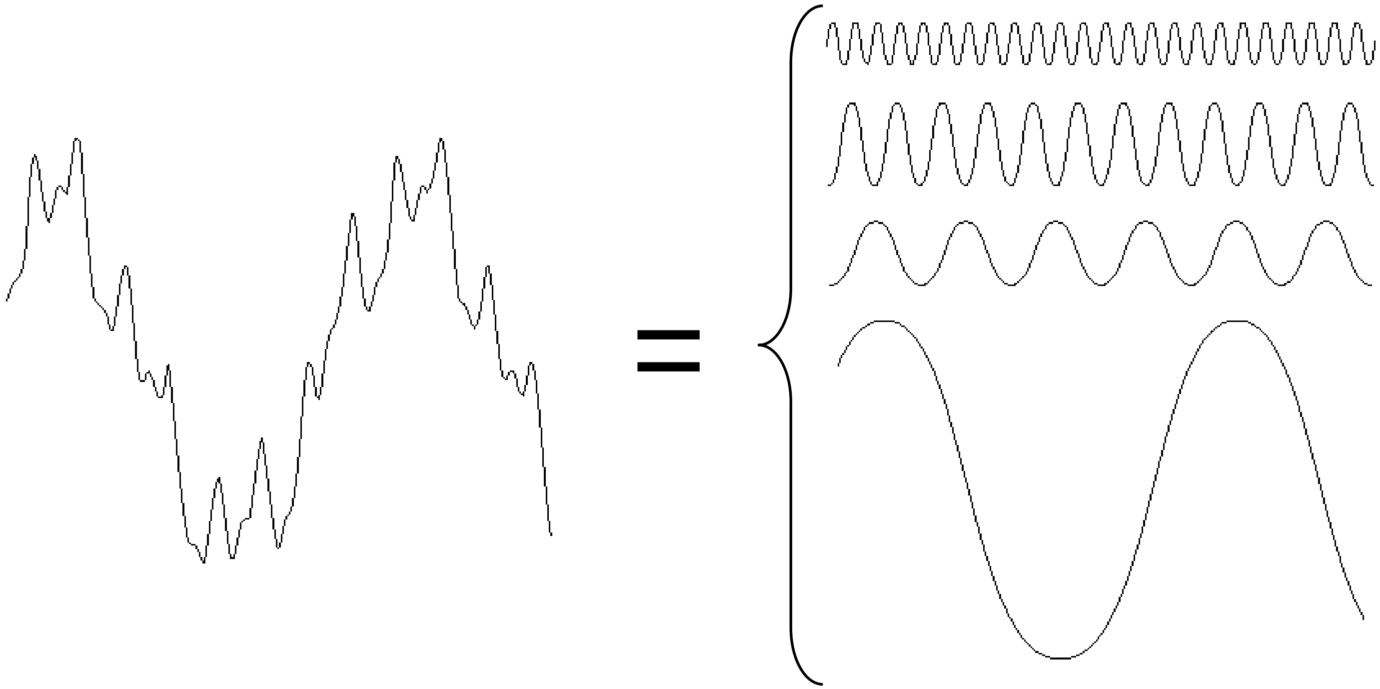
- Jean Baptiste Joseph Fourier
- The Fourier series & the Fourier transform
- Image Processing in the frequency domain
  - Image smoothing
  - Image sharpening
- Fast Fourier Transform

Jean Baptiste Joseph Fourier

- Fourier was born in Auxerre, France in 1768
  - Most famous for his work "*La Théorie Analitique de la Chaleur*" published in 1822
  - Translated into English in 1878: "*The Analytic Theory of Heat*"
- Nobody paid much attention when the work was first published
- One of the most important mathematical theories in modern engineering

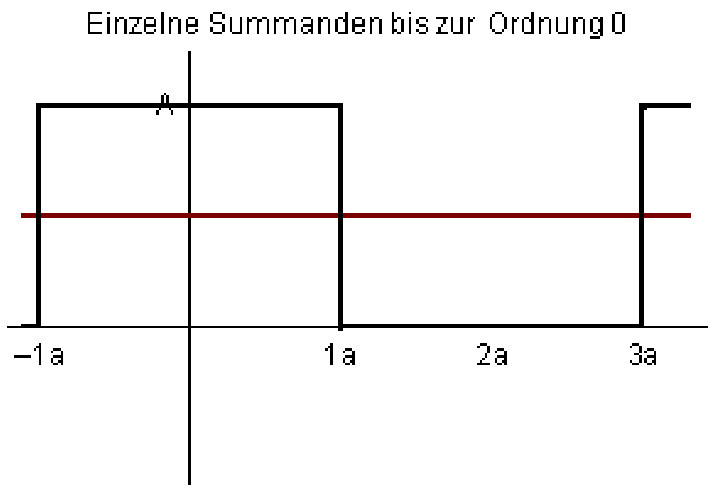
The Big Idea

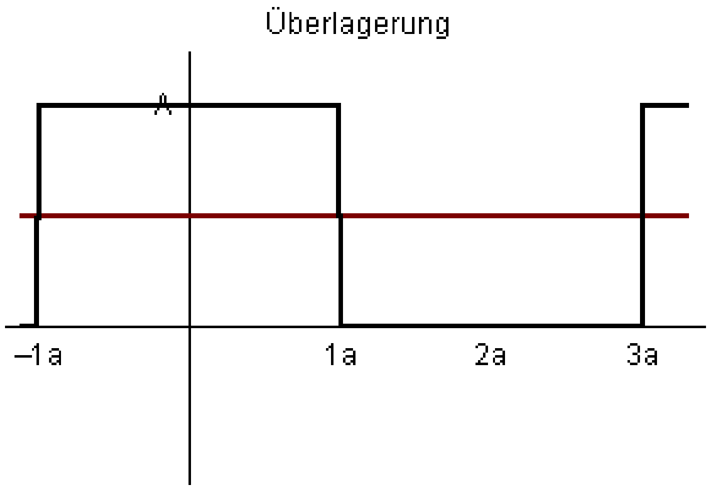
Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient – a *Fourier series*

The Big Idea (cont…)

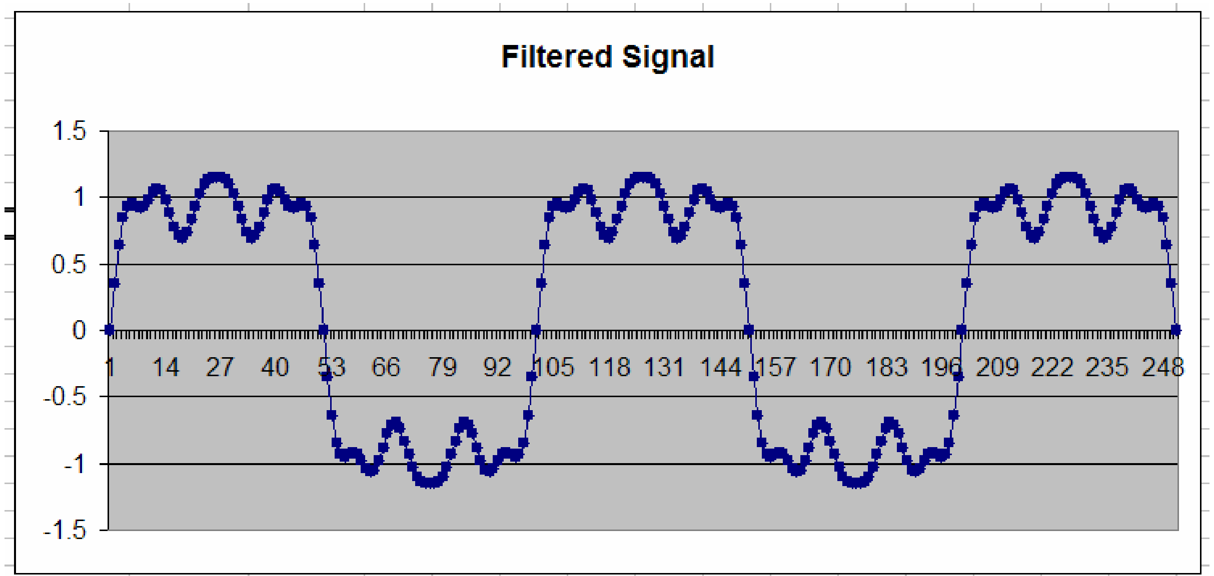


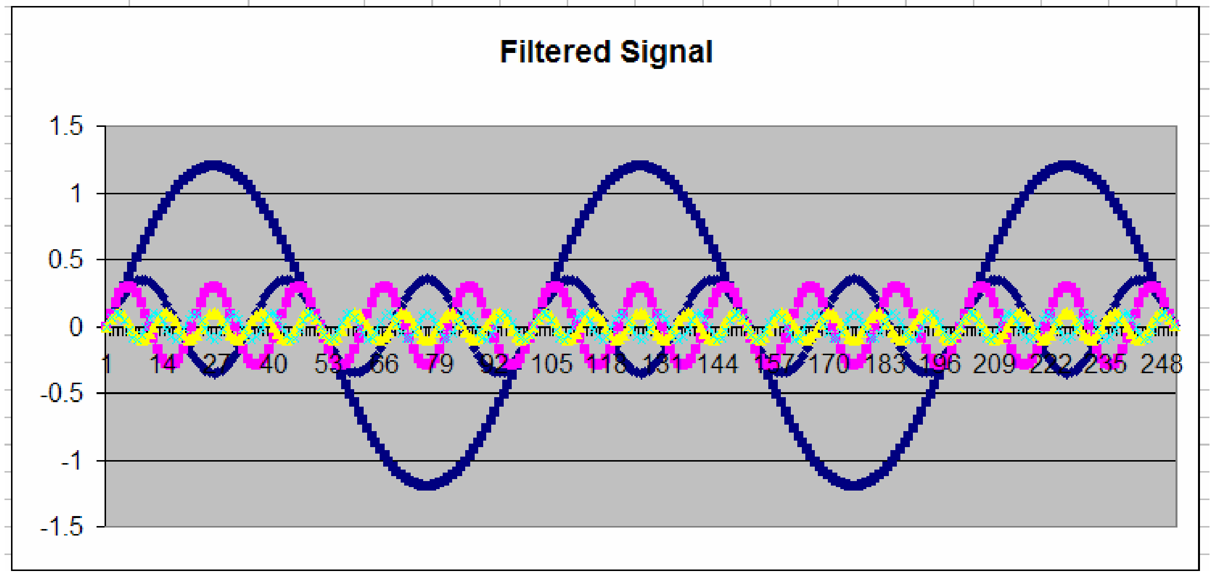


Notice how we get closer and closer to the original function as we add more and more frequencies

Taken from www.tfh-berlin.de/~schwenk/hobby/fourier/Welcome.html

The Big Idea (cont…)





Frequency domain signal processing example in Excel

Fourier Transform Image Processing

- Any periodic object can be represented by a summation of a series of cosine waves
- The Operation of Fourier transformation of an image replaces the image (real space) be a series of amplitudes and frequencies of the cosine waves that make it up
- Fourier space is also referred to as frequency space
- If there are repeats in the structure at specific frequencies, these will appear as peaks in Fourier space

The Discrete Fourier Transform (DFT)

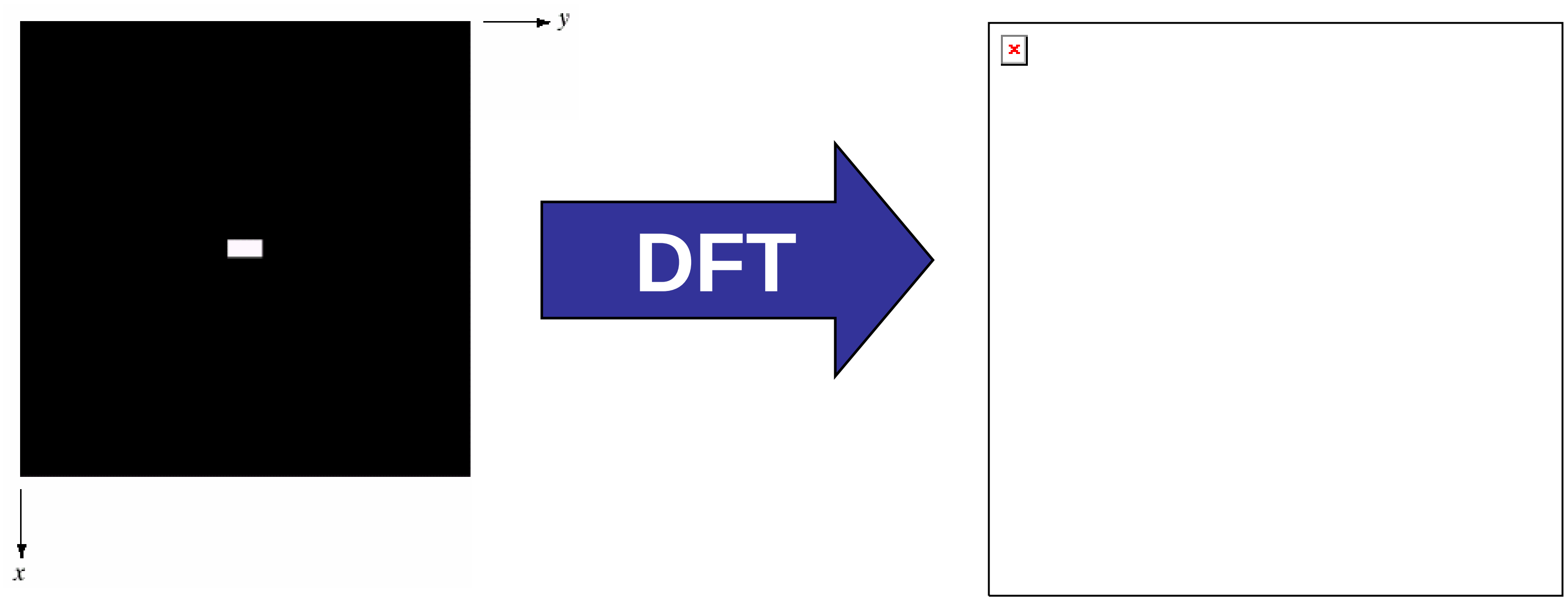
The *Discrete Fourier Transform* of $f(x, y)$, for $x = 0, 1, 2\ldots M-1$ and $y = 0,1,2\ldots N-1$, denoted by $F(u, v)$, is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

for $u = 0, 1, 2\ldots M-1$ and $v = 0, 1, 2\ldots N-1$.

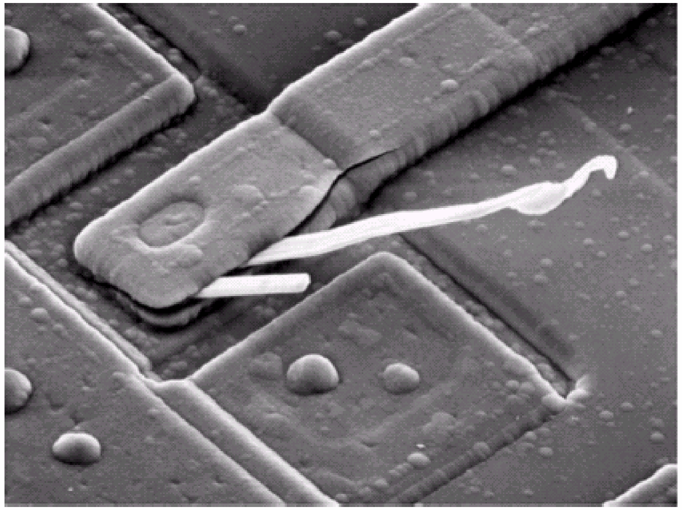
DFT & Images

The DFT of a two dimensional image can be visualised by showing the spectrum of the images component frequencies

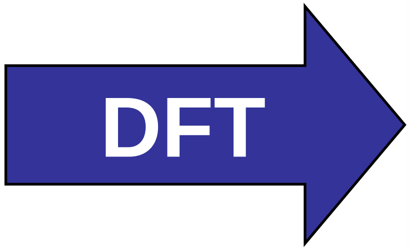


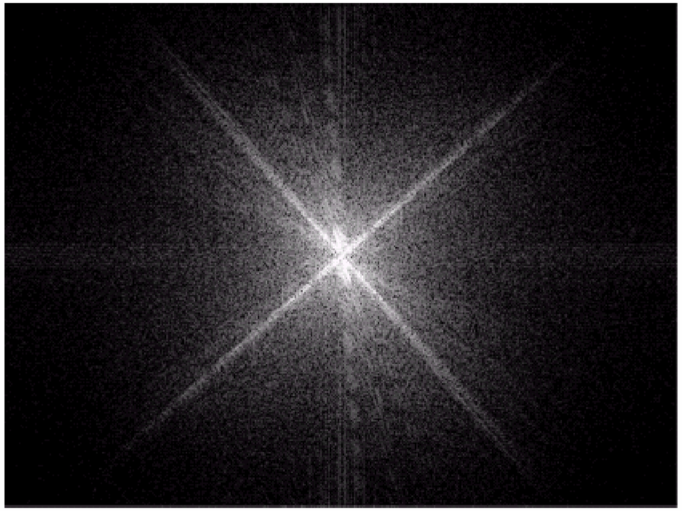
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

DFT & Images (cont…)



Scanning electron microscope image of an integrated circuit magnified ~2500 times





Fourier spectrum of the image

Features from an image can even sometimes be seen in the Fourier spectrum of the image

The Inverse DFT

It is really important to note that the Fourier transform is completely **reversible**

The inverse DFT is given by:

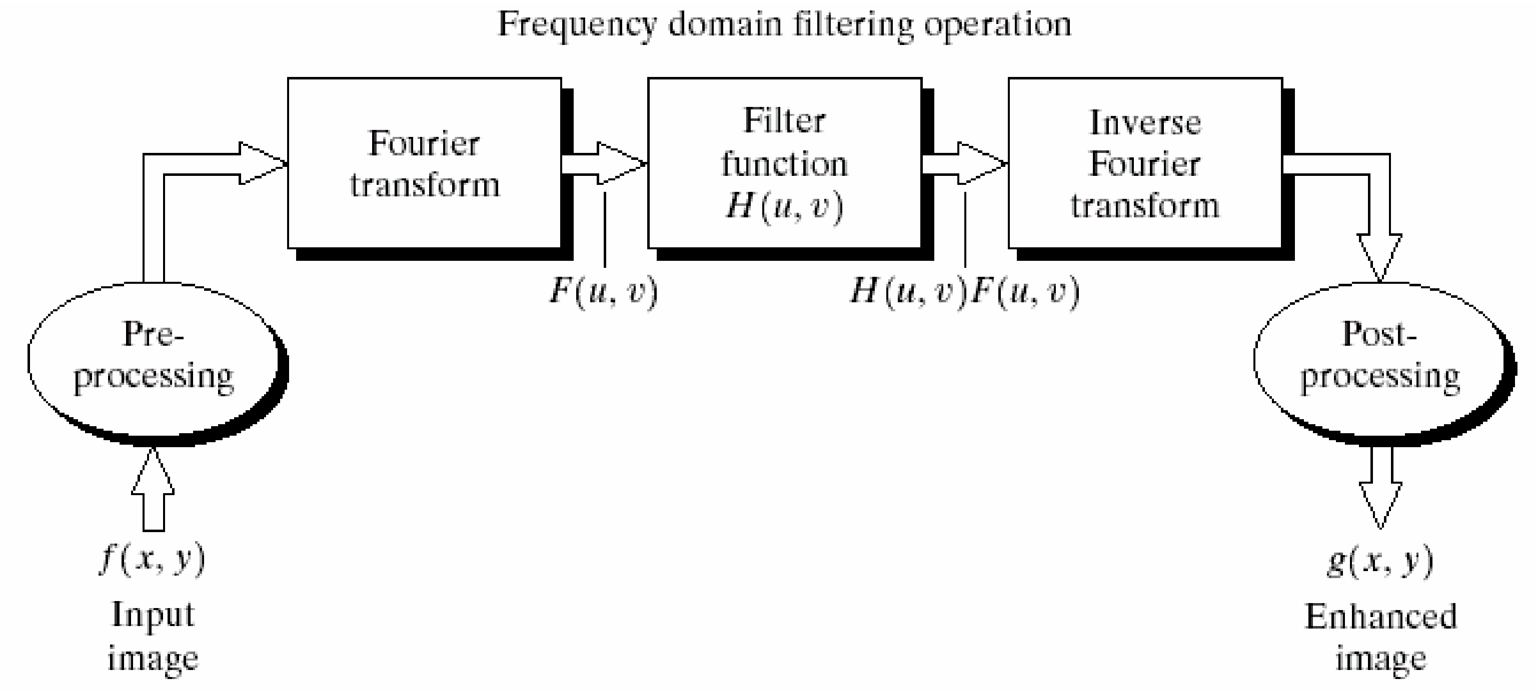
$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$$

for x = 0, 1, 2…M-1 and y = 0, 1, 2…N-1

The DFT and Image Processing

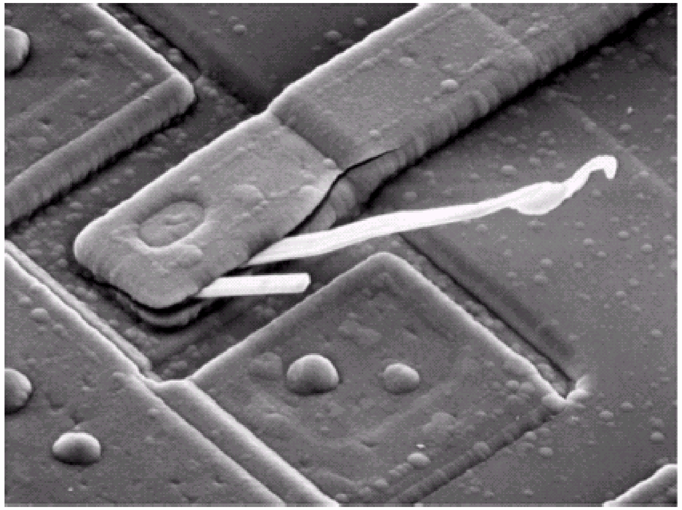
To filter an image in the frequency domain:

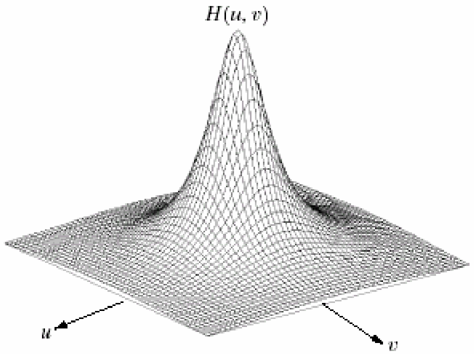
1. Compute $F(u,v)$ the DFT of the image
2. Multiply $F(u,v)$ by a filter function $H(u,v)$
3. Compute the inverse DFT of the result

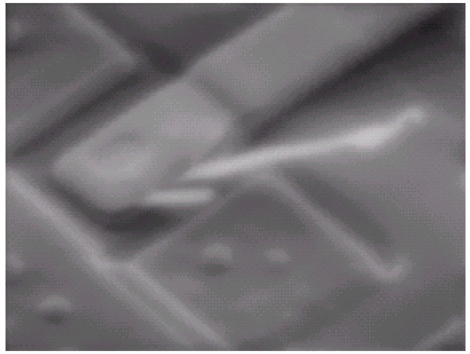


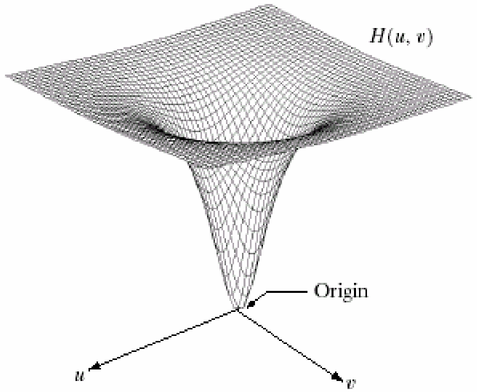
Some Basic Frequency Domain Filters

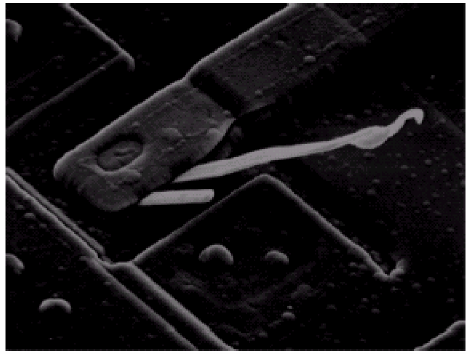
Low Pass Filter











High Pass Filter

Smoothing Frequency Domain Filters

Smoothing is achieved in the frequency domain by dropping out the high frequency components

The basic model for filtering is:

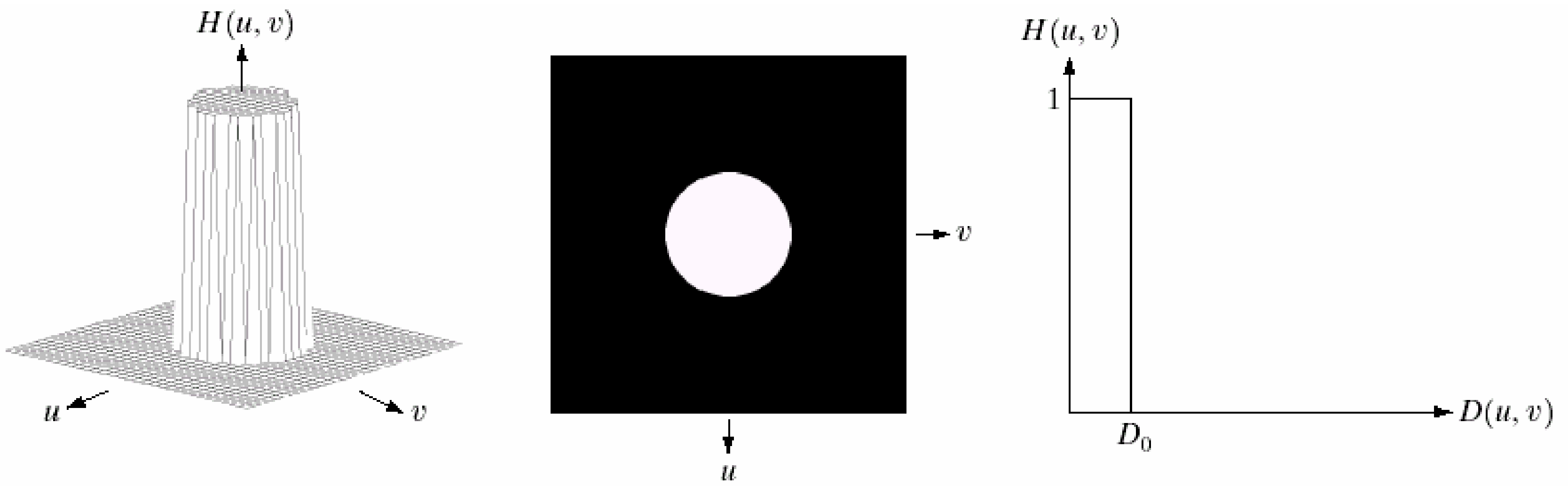
$$G(u,v) = H(u,v)F(u,v)$$

where $F(u,v)$ is the Fourier transform of the image being filtered and $H(u,v)$ is the filter transform function

Low pass filters – only pass the low frequencies, drop the high ones

Ideal Low Pass Filter

Simply cut off all high frequency components that are a specified distance D_0 from the origin of the transform



changing the distance changes the behaviour of the filter

Ideal Low Pass Filter (cont…)

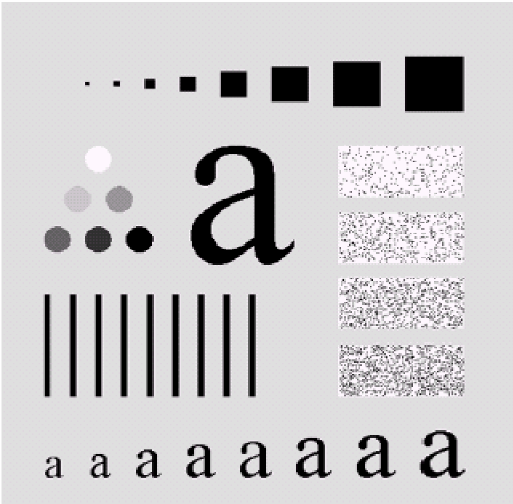
The transfer function for the ideal low pass filter can be given as:

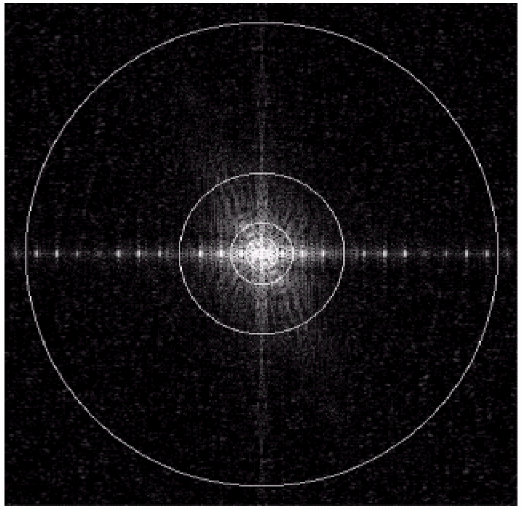
$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ is given as:

$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

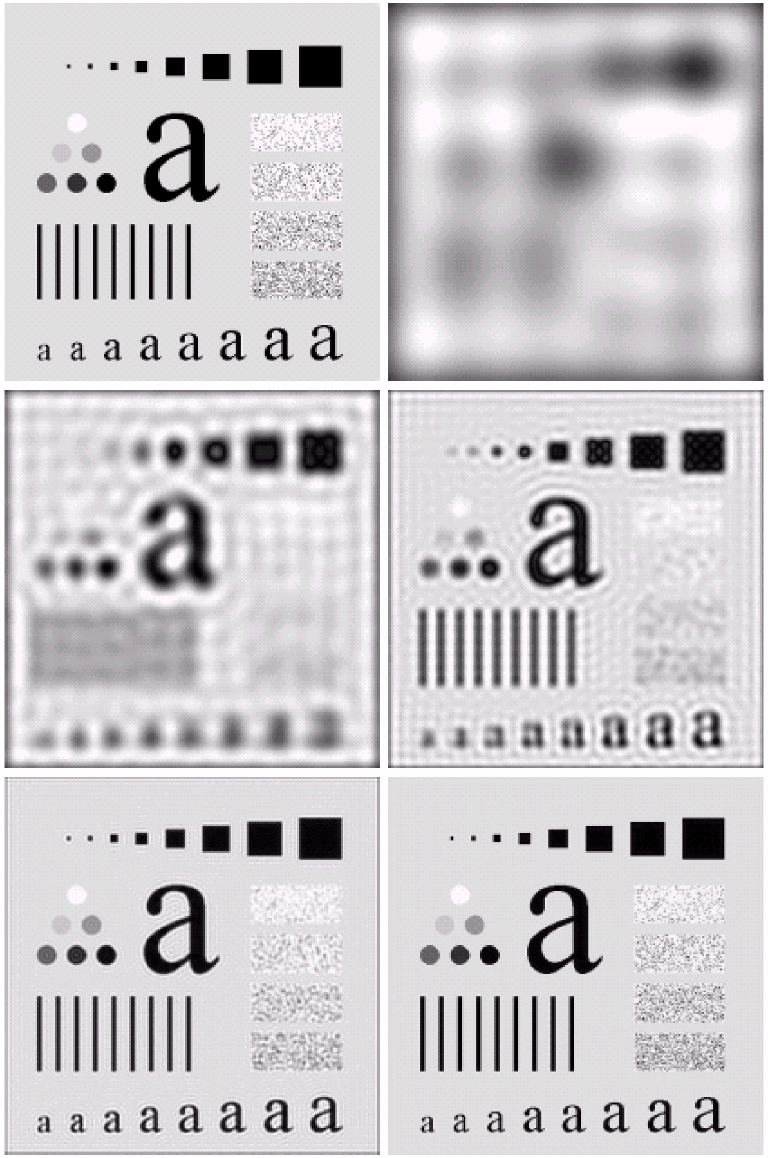
Ideal Low Pass Filter (cont…)





Above we show an image, it's Fourier spectrum and a series of ideal low pass filters of radius 5, 15, 30, 80 and 230 superimposed on top of it

Ideal Low Pass Filter (cont…)



Original image

Result of filtering with ideal low pass filter of radius 5

Result of filtering with ideal low pass filter of radius 15

Result of filtering with ideal low pass filter of radius 30

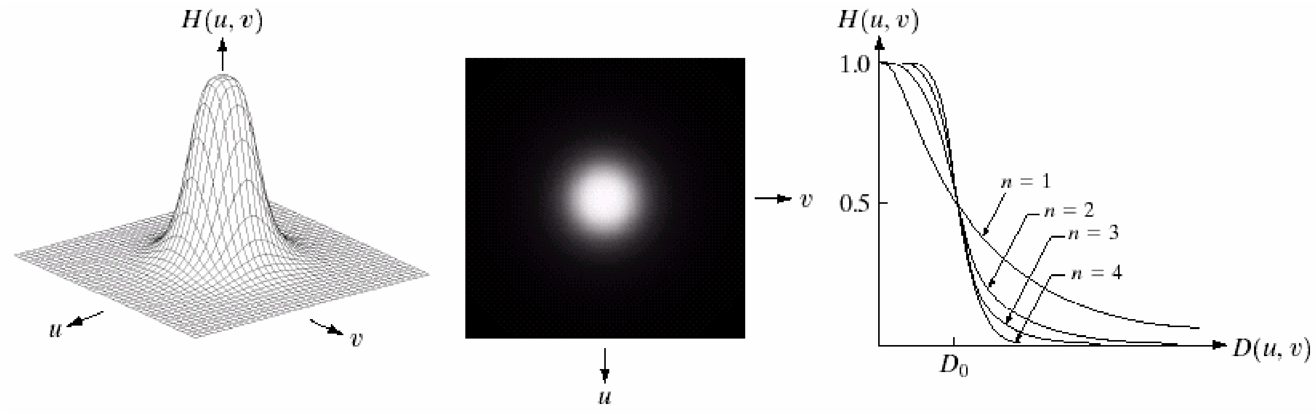
Result of filtering with ideal low pass filter of radius 80

Result of filtering with ideal low pass filter of radius 230

Butterworth Lowpass Filters

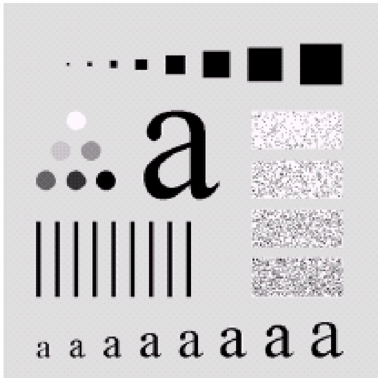
The transfer function of a Butterworth lowpass filter of order n with cutoff frequency at distance D_0 from the origin is defined as:

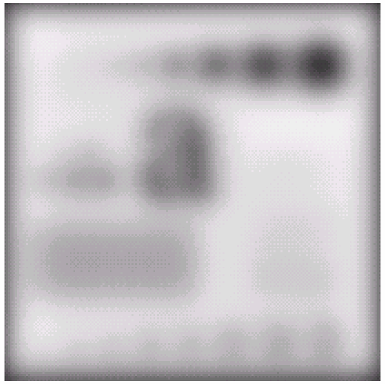
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



Butterworth Lowpass Filter (cont…)

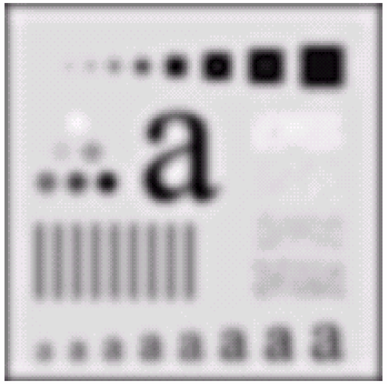
Original image

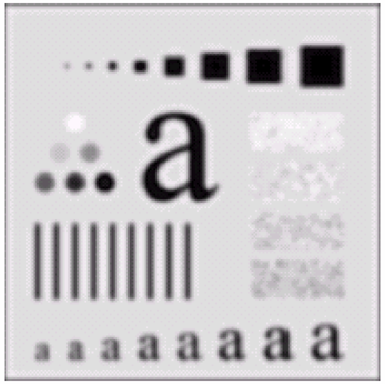




Result of filtering with Butterworth filter of order 2 and cutoff radius 5

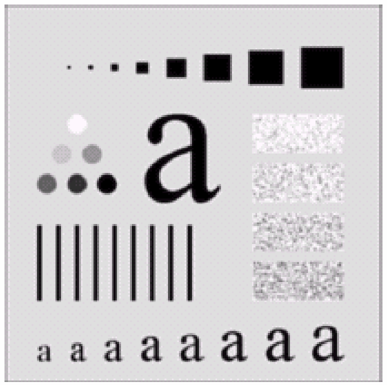
Result of filtering with Butterworth filter of order 2 and cutoff radius 15

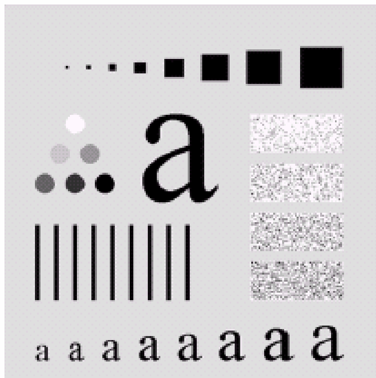




Result of filtering with Butterworth filter of order 2 and cutoff radius 30

Result of filtering with Butterworth filter of order 2 and cutoff radius 80



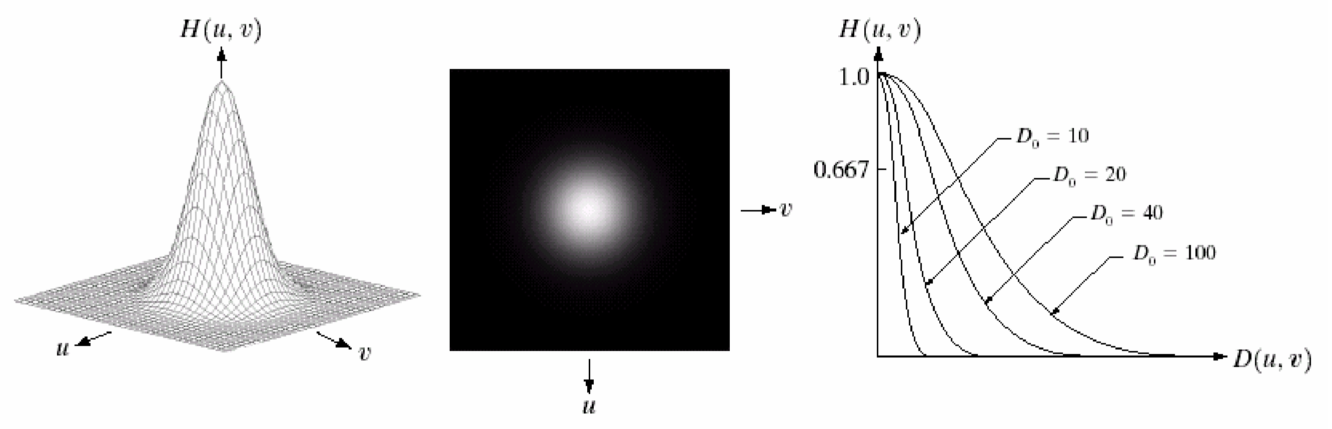


Result of filtering with Butterworth filter of order 2 and cutoff radius 230

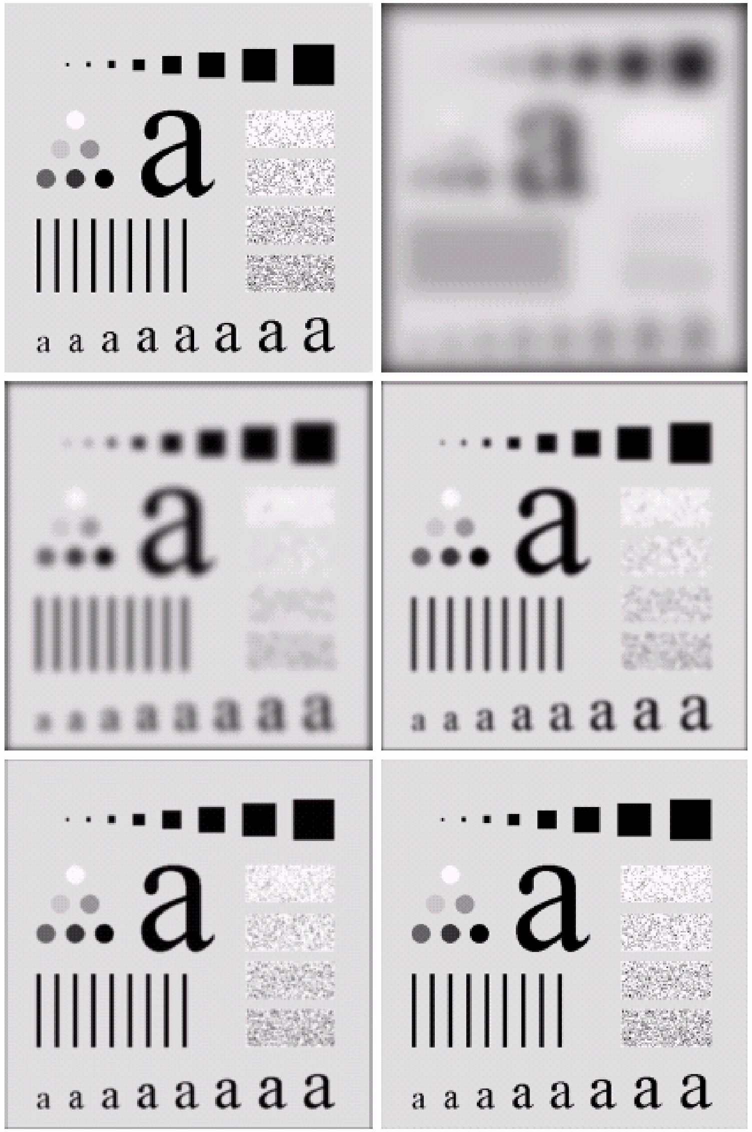
Gaussian Lowpass Filters

The transfer function of a Gaussian lowpass filter is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



Gaussian Lowpass Filters (cont…)



Original image

Result of filtering with Gaussian filter with cutoff radius 5

Result of filtering with Gaussian filter with cutoff radius 15

Result of filtering with Gaussian filter with cutoff radius 30

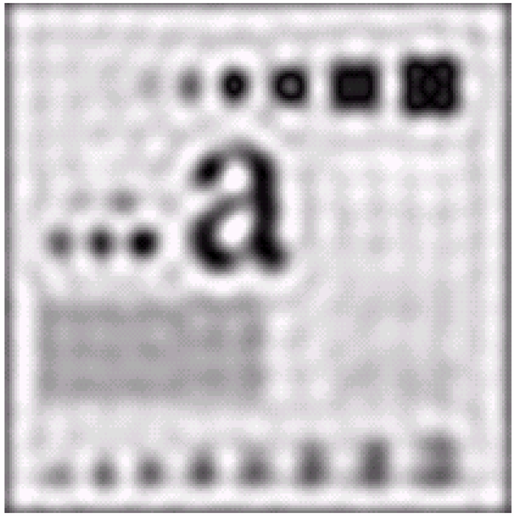
Result of filtering with Gaussian filter with cutoff radius 85

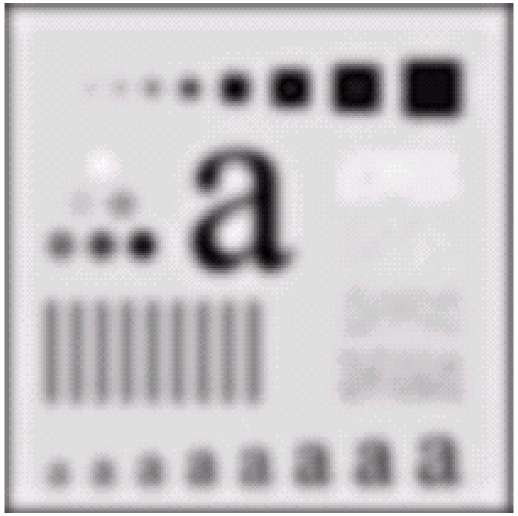
Result of filtering with Gaussian filter with cutoff radius 230

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Lowpass Filters Compared

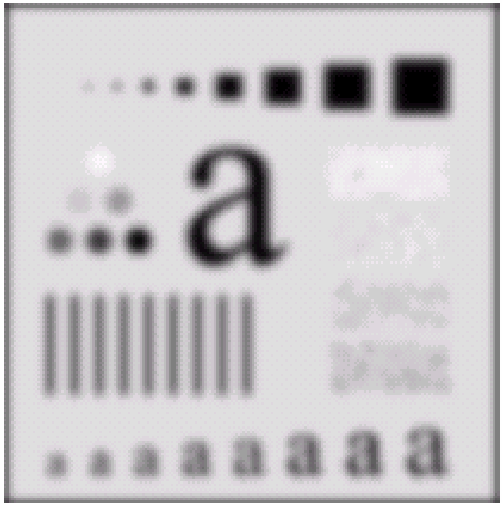
Result of filtering with ideal low pass filter of radius 15





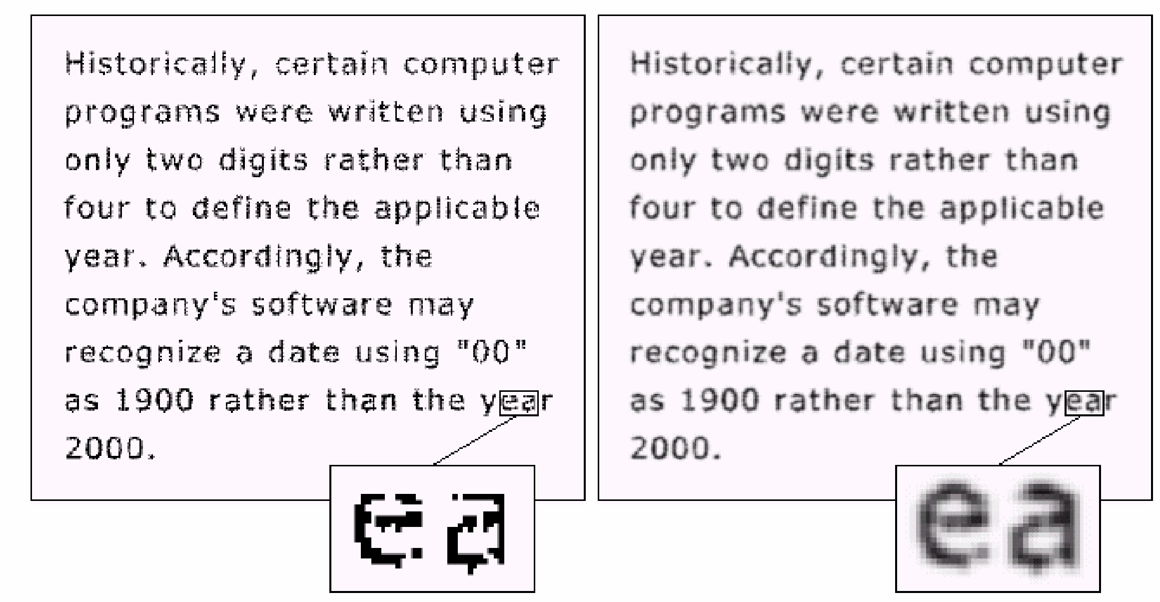
Result of filtering with Butterworth filter of order 2 and cutoff radius 15

Result of filtering with Gaussian filter with cutoff radius 15



Lowpass Filtering Examples

A low pass Gaussian filter is used to connect broken text

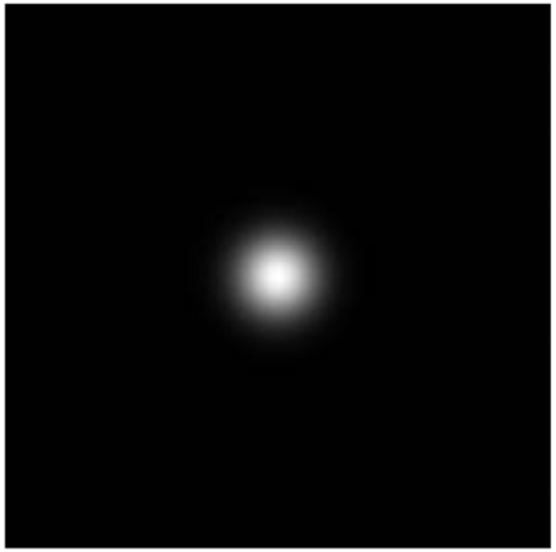


Lowpass Filtering Examples (cont…)

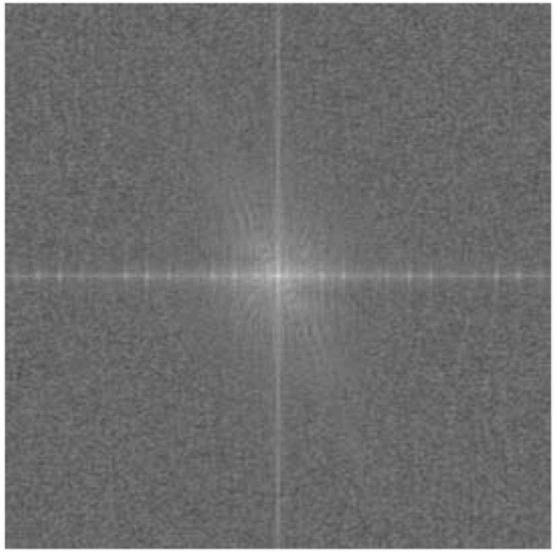
Different lowpass Gaussian filters used to remove blemishes in a photograph

Lowpass Filtering Examples (cont…)

Original image



Gaussian lowpass filter



Spectrum of original image

Processed image

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Sharpening in the Frequency Domain

Edges and fine detail in images are associated with high frequency components

High pass filters – only pass the high frequencies, drop the low ones

High pass frequencies are precisely the reverse of low pass filters, so:

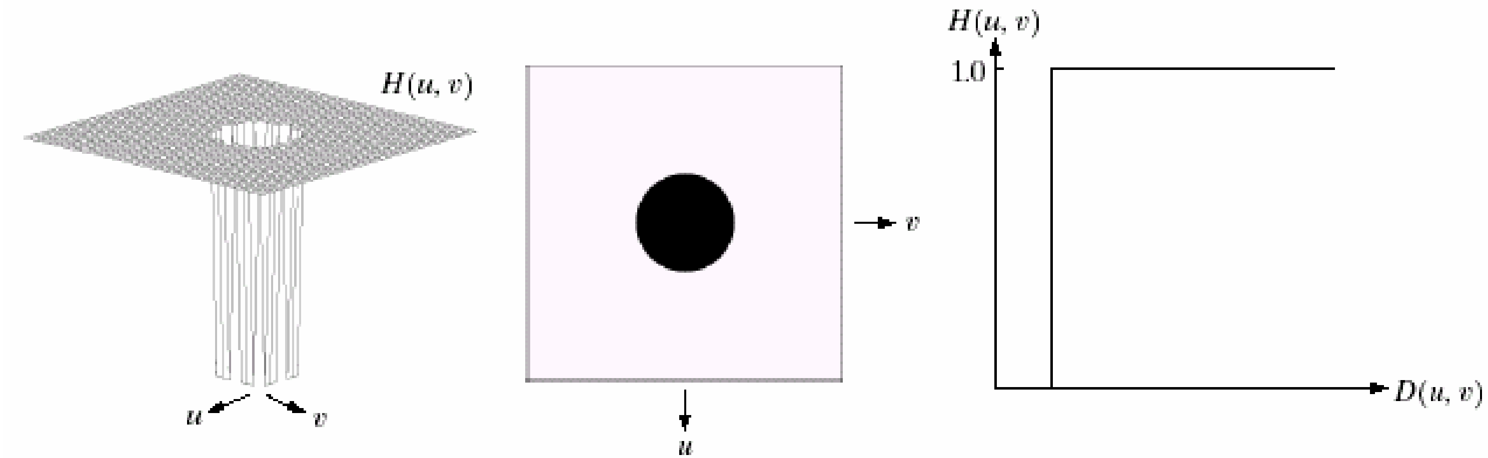
$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

Ideal High Pass Filters

The ideal high pass filter is given as:

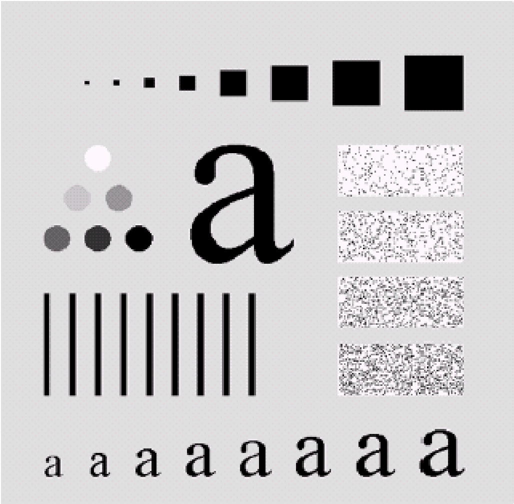
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

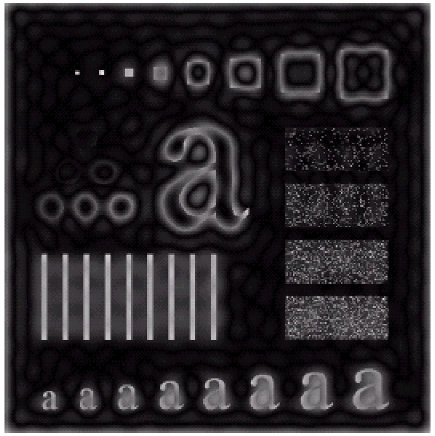
where D_0 is the cut off distance as before



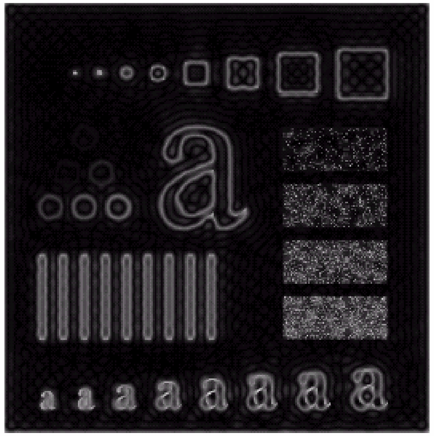
Ideal High Pass Filters (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

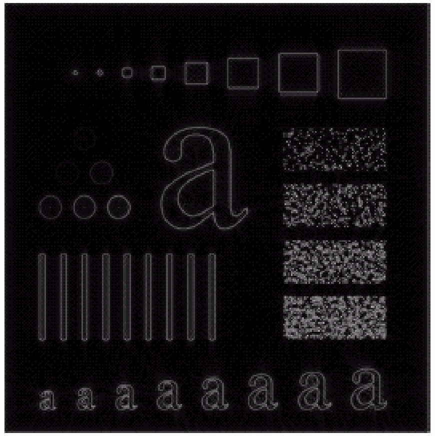




Results of ideal high pass filtering with $D_0 = 15$



Results of ideal high pass filtering with $D_0 = 30$



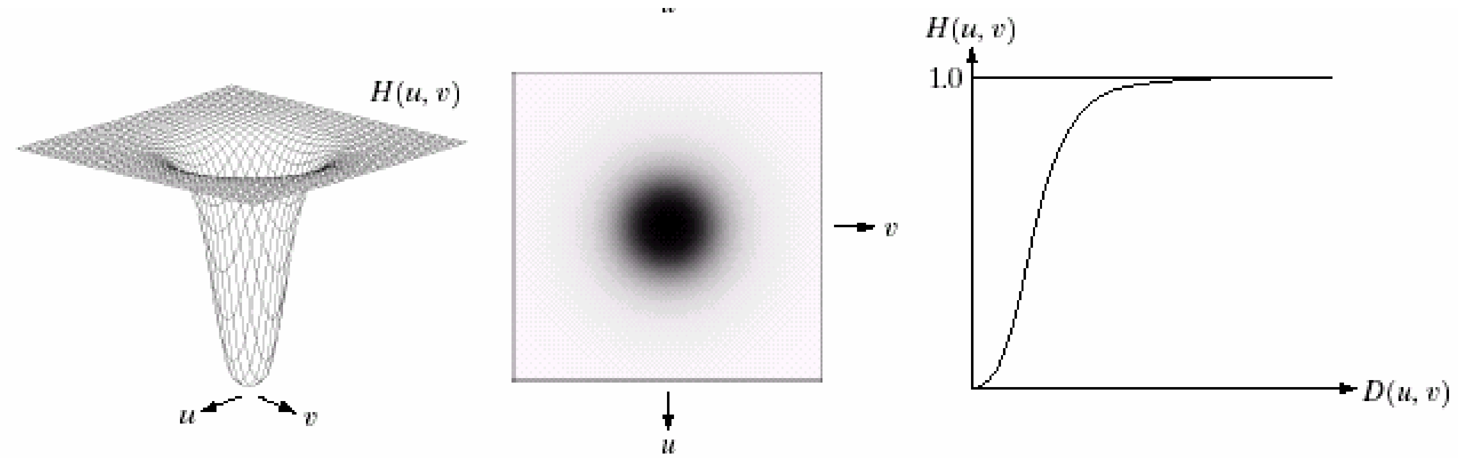
Results of ideal high pass filtering with $D_0 = 80$

Butterworth High Pass Filters

The Butterworth high pass filter is given as:

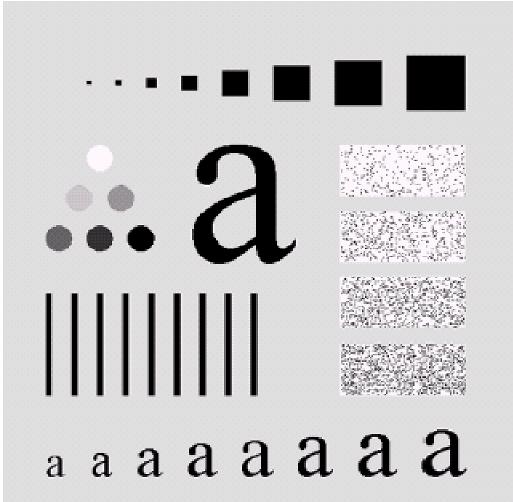
$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

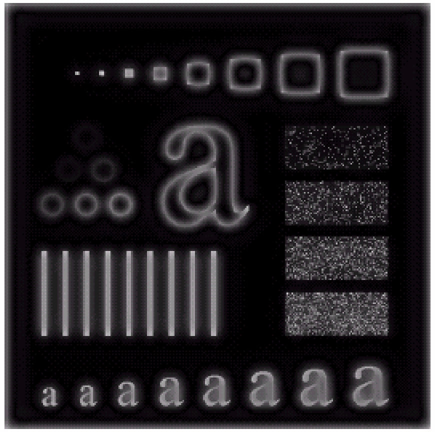
where n is the order and D_0 is the cut off distance as before



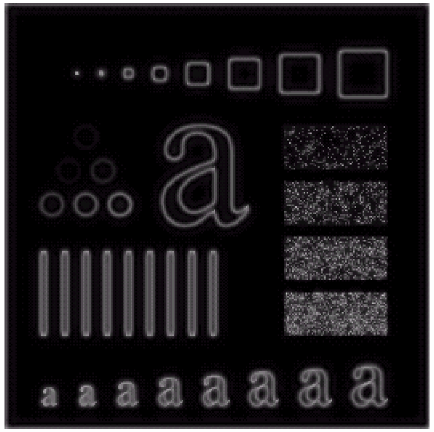
Butterworth High Pass Filters (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

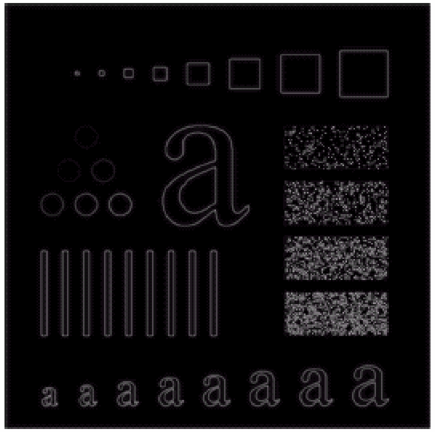




Results of Butterworth high pass filtering of order 2 with $D_0 = 15$



Results of Butterworth high pass filtering of order 2 with $D_0 = 30$



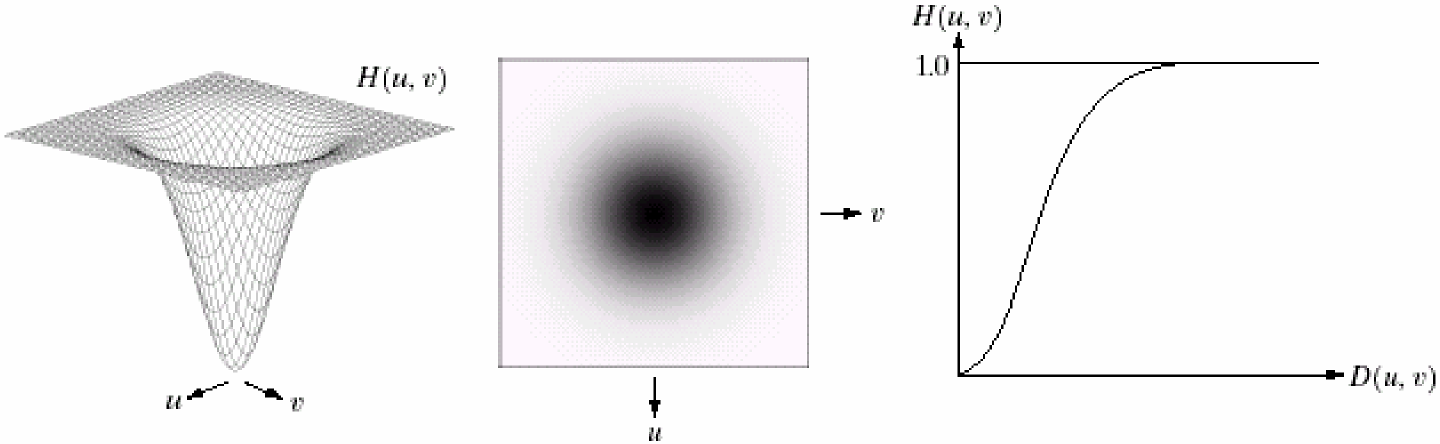
Results of Butterworth high pass filtering of order 2 with $D_0 = 80$

Gaussian High Pass Filters

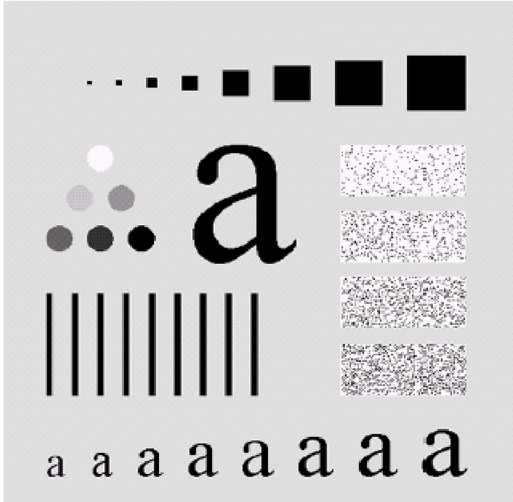
The Gaussian high pass filter is given as:

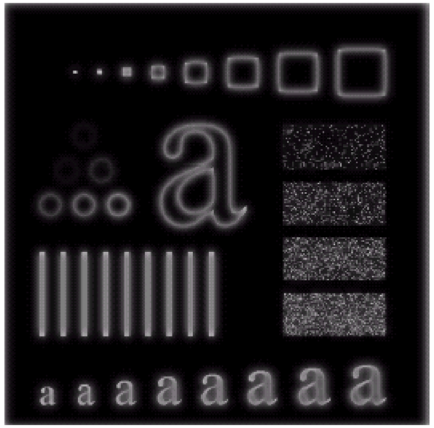
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

where D_0 is the cut off distance as before

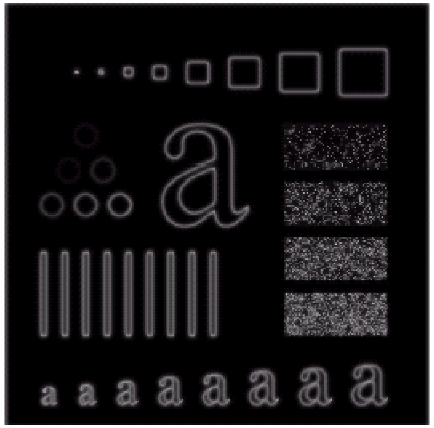


Gaussian High Pass Filters (cont…)





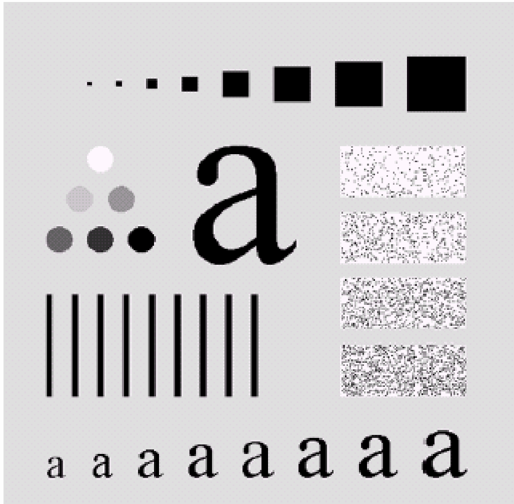
Results of Gaussian high pass filtering with $D_0 = 15$

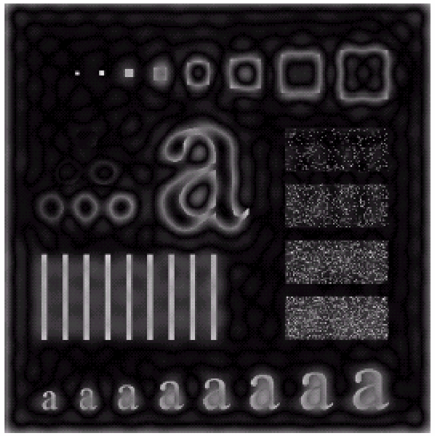


Results of Gaussian high pass filtering with $D_0 = 30$

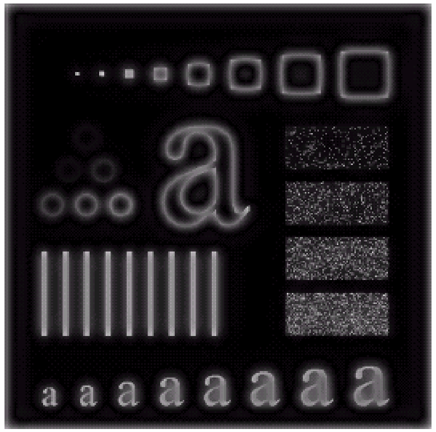
Results of Gaussian high pass filtering with $D_0 = 80$

Highpass Filter Comparison

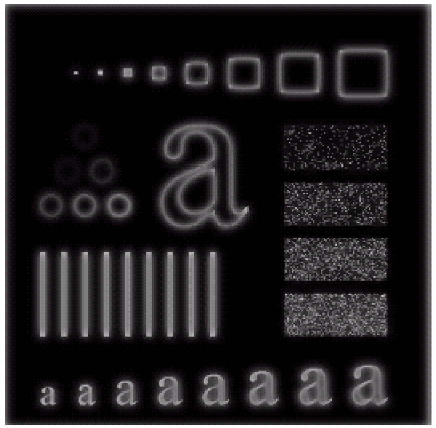




Results of ideal high pass filtering with $D_0 = 15$

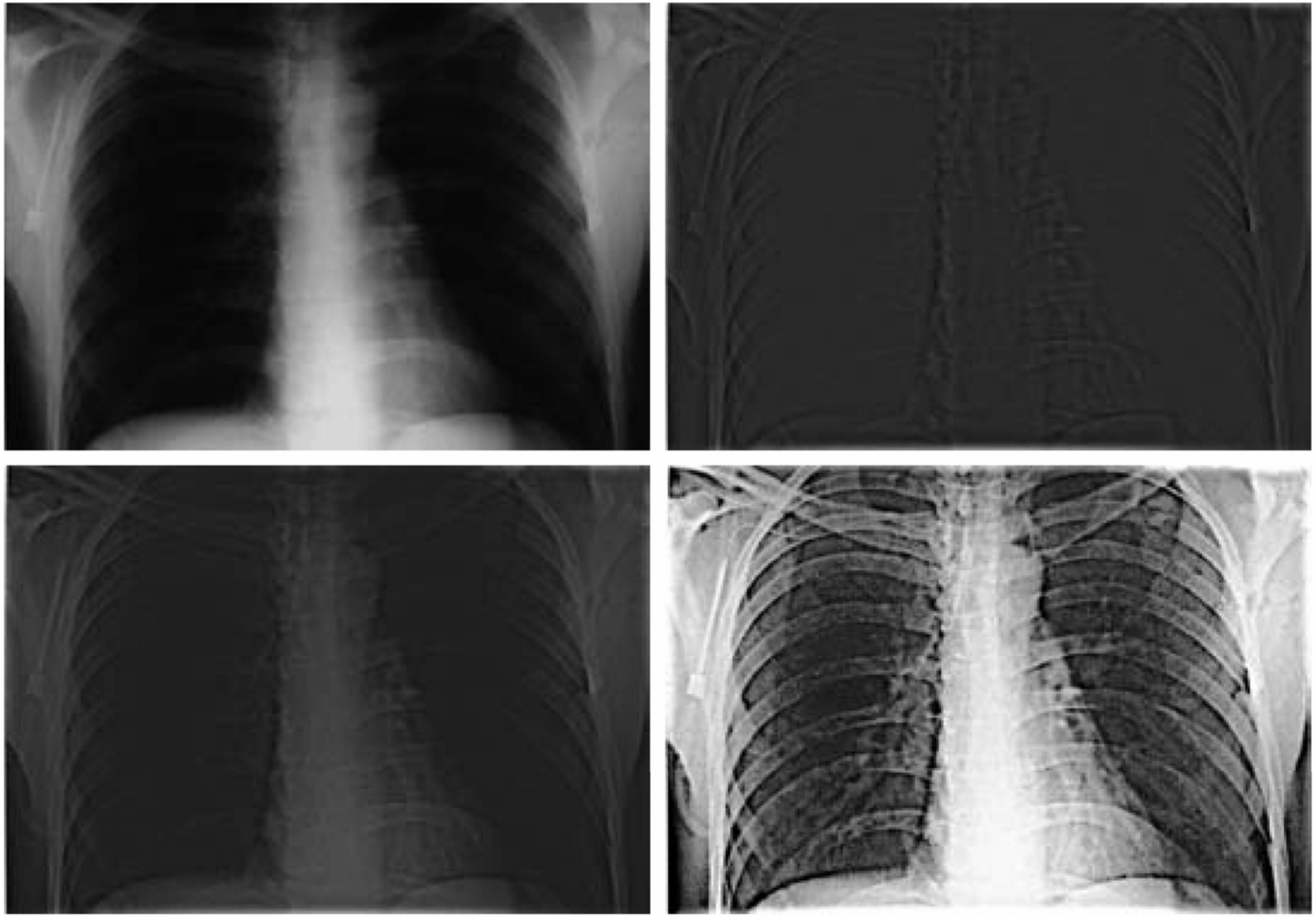


Results of Butterworth high pass filtering of order 2 with $D_0 = 15$



Results of Gaussian high pass filtering with $D_0 = 15$

Highpass Filtering Example



Original image

Highpass filtering result

High frequency emphasis result

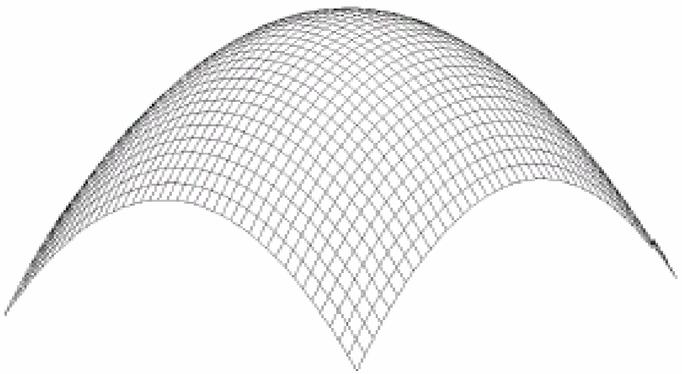
After histogram equalisation

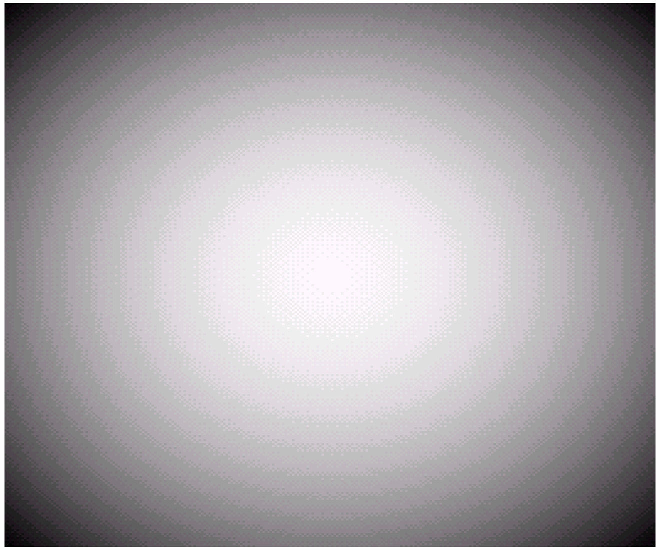
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Laplacian In The Frequency Domain

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

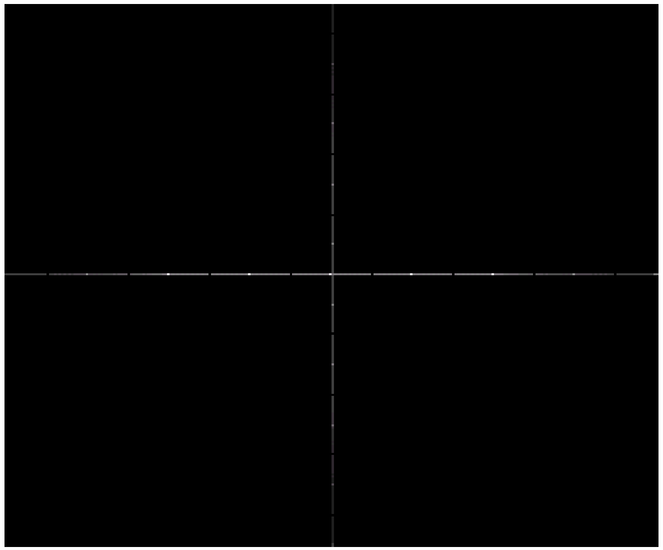
Laplacian in the frequency domain

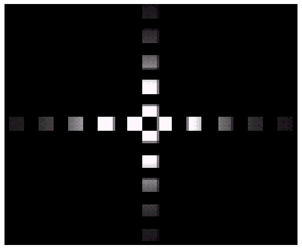


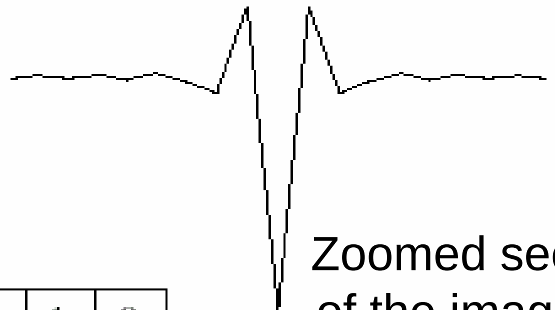


2-D image of Laplacian in the frequency domain

Inverse DFT of Laplacian in the frequency domain





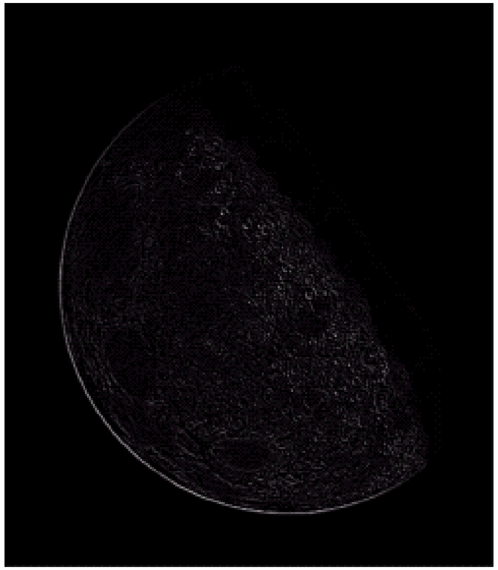


0	1	0
1	−4	1
0	1	0

Zoomed section of the image on the left compared to spatial filter

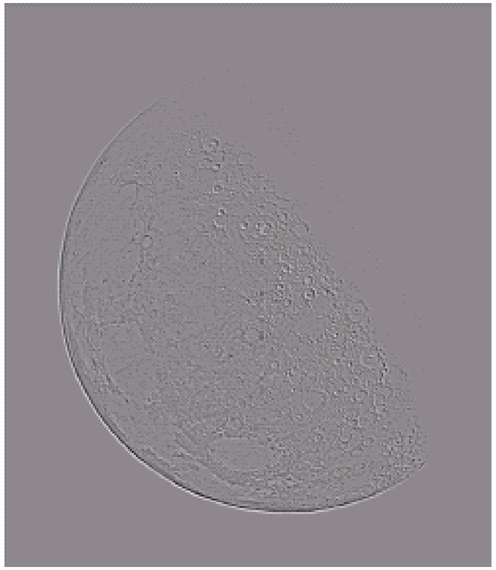
Frequency Domain Laplacian Example

Original image



Laplacian filtered image

Laplacian image scaled



Enhanced image

Fast Fourier Transform

The reason that Fourier based techniques have become so popular is the development of the *Fast Fourier Transform (FFT)* algorithm

Allows the Fourier transform to be carried out in a reasonable amount of time

Reduces the amount of time required to perform a Fourier transform by a factor of 100 – 600 times!

Dangers of Fourier transforms

Can introduce periodicities where none are present

Edge effects