Machine Vision

Lecture 7

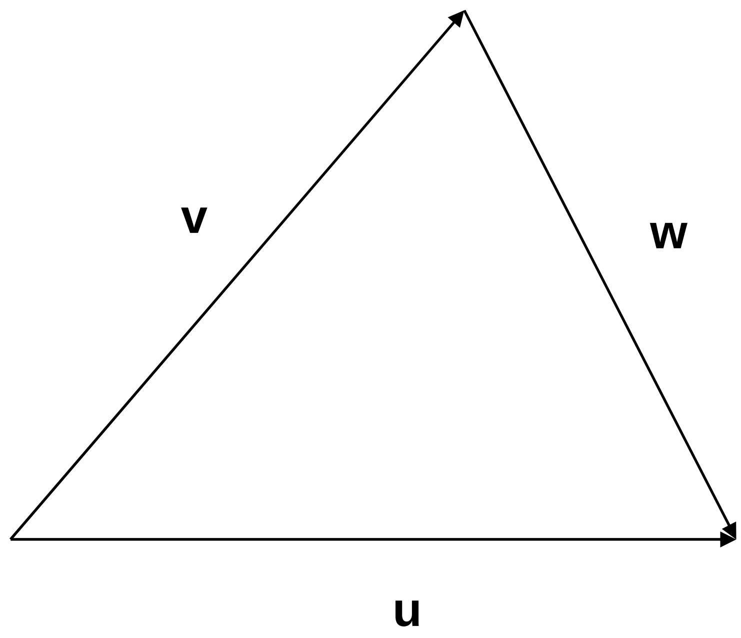
Transformations of images.

Vectors

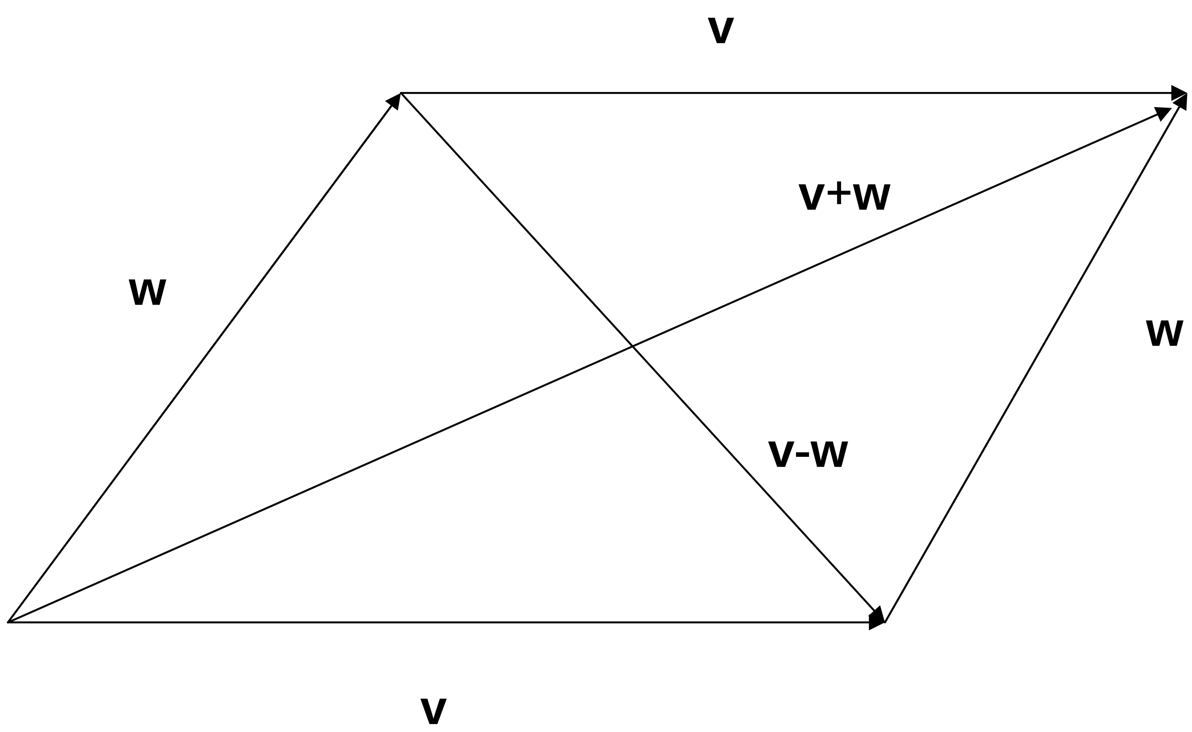
- Vector **v** is an entity with magnitude (length) and direction
- Vector with magnitude 1 is normalized vector
- Not that does not have a location
- Two vectors with same magnitude and direction are equal, no matter where draw on the page (screen)

Vector addition

- **u = v+w**



Vector addition and subtruction



Real Vector Space

- Vector space allows to represent vectors symbolically
- Real vector space is the set of all ordered pairs of real numbers

- 2-dimensinal $\Re^2 = \{(x, y) \mid x, y \in \Re\}$
- 3-dimensinal $\Re^3 = \{(x, y, z) \mid x, y, z \in \Re\}$

...

Standard basis for a vector space

$$\mathbf{e}_0 = (1, 0, ..., 0)$$
$$\mathbf{e}_1 = (0, 1, ..., 0)$$
$$...$$
$$\mathbf{e}_{n-1} = (0, 0, ..., 1)$$

Main property of basis β is that for every vector **v** in V there is a unique linear combination of the vectors in β that equal **v**.

$$\mathbf{v} = a_0\mathbf{b}_0 + a_1\mathbf{b}_1 + ... + a_{n-1}\mathbf{b}_{n-1}$$

or for 3D vector space

$$\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

We can think of *x,y* and *z* as the amounts we move in the **I, j** and **k** directions.

Inner product

- Set of functions <**v,w**> returning a real scalar with following properties:

  <**v,w**> = <**w,v**>

  <**u**+**v,w**> = <**u,w**> + <**v,w**>

  a<**v,w**> = <a**v,w**>

  <**v,v**> >= 0

  <**v,v**> = 0 iff **v** = 0

Dot product

$\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||||\mathbf{w}|| \cos\theta$

$\mathbf{v} \cdot \mathbf{w} = u_x w_x + u_y w_y + u_z w_z$

If $\mathbf{v} \cdot \mathbf{w} = 0$ then vectors **v** and **w** are orthogonal

Cross product

- Aim is determination new vector orthogonal to both determined vectors
- Also it is known as *vector product*

$$\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\| \sin\theta$$

and $\mathbf{v} \times \mathbf{w} = -(\mathbf{w} \times \mathbf{v})$

$$\mathbf{v} \times \mathbf{w} = (v_y w_z - w_y v_z,\ y_z w_x - w_z v_x,\ v_x w_y - w_x v_y)$$

Triple products

- Vector triple product

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$$

- Scalar triple product

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

- If $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) > 0$ then the shortest rotation from v to w is in counterclockwise direction, otherwise – in clockwise direction

Affine space

- An affine space consists of a set of points W and a vector space V
- Relation between the points and vectors:
  - For every pair of points P and Q in W exist a unique vector **v** in V such that

$$\mathbf{v} = Q - P$$

  - For every point P in W and every vector **v** in V exist a unique point Q such that

$$Q = P + \mathbf{v}$$

Representation of any point in W is P = O + **v,** where

O is fixed point in W named as *origin*

Combination of the origin O and basis vectors is known as a *coordinate frame*

If we work with standard origin (0,0,..,0) a standard basis then we work with *Cartesian frame*

Polar and spherical coordinates

- Relation between polar and Cartesian coordinates

$$x = r \cos \theta,$$
$$y = r \sin \theta$$

- Relation between spherical and Cartesian coordinates

$$X = \rho \sin \varphi \cos \theta,$$
$$Y = \rho \sin \varphi \sin \theta,$$
$$Z = \rho \cos \varphi,$$

where φ is angle between z-axis and projection of **v** on yz plane, θ is angle between x-axis and projection of **v** on xy plane

Triangle

- Determined by 3 vectors

$$\mathbf{v}_0 = P_1 - P_2$$

$$\mathbf{v}_1 = P_2 – P_1$$

$$\mathbf{v}_3 = P_0 – P_2$$

Linear transformations

- Only operations possible in linear transformations are multiplication by a constant and addition
- Linear transformation *T* is a mapping between two vector spaces V and W, where for all **v** in V and all scalars a:
  - $T(\mathbf{v}_0+\mathbf{v}_1) = T(\mathbf{v}_0)+T(\mathbf{v}_1)$ for all $\mathbf{v}_0,\mathbf{v}_1$ in V,
  - $T(a\mathbf{v}) = aT(\mathbf{v})$ for all **v** in V

Matrices

- It is 2-dimensional array of value (n x m)
- Elements, rows and columns
- Diagonal and trace
- Square, zero, diagonal and triangular matrix

Simple operations

- Addition

$$\mathbf{S} = \mathbf{A} + \mathbf{B}$$

$$s_{i,j} = a_{i,j} + b_{i,j}$$

- Scalar multiplication

$$\mathbf{P} = s\mathbf{A}$$

- Transpose

$$\mathbf{A}^T \quad \begin{bmatrix} 2,-1 \\ 0,2 \\ 6,3 \end{bmatrix} = \begin{bmatrix} 2,0,6 \\ -1,2,3 \end{bmatrix}$$

Matrices (cont.)

- Vector may be represented by matrix with one column (or row)
- Block matrix or matrix with submatrices

$$\begin{bmatrix} 2,3,0 \\ -3,2,0 \\ 0,0,1 \end{bmatrix} \text{may be presented as}$$

$$\begin{bmatrix} \mathbf{A},0 \\ o^T,1 \end{bmatrix} \text{where } A = \begin{bmatrix} 2,3 \\ -3,2 \end{bmatrix}$$

Matrix product

C = AB

$$c_{i,j} = \sum_{k=0}^{n-1} a_{i,k} b_{k,j}$$

It is used for representation of linear transformation of vector

B = Ax,

where x is n-dimensional vector and b is m-dimensional vector (result of transformation)

Affine transformations

- For points P and constants *a* in affine space A

$T(a_0 P_0 + \ldots + a_{n-1} P_{n-1}) = a_0 T(P_0) + \ldots + a_{n-1} T P_{n-1}$

where $a_0 + \ldots + a_{n-1} = 1$

Affine transformations provide remain of collinearity and coplanarity during transformations

Transformation is $V_{new} = V_{old} \times M$

Affine transformations (cont.)

- Translation
  - T(P) = P + **t**
  - Simulation of moving of object in space, size and shape of object are not changed
- Matrix for translation of vector

[1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[x y z 1]

Affine transformations (cont.)

Rotation

- Around x-axis

  y’ = ycosθ - zsinθ

  z’ = ycosθ + zcosθ

$$\mathbf{R_x} = \begin{bmatrix} 1,0,0 \\ 0,\cos\theta,-\sin\theta \\ 0,\sin\theta,\cos\theta \end{bmatrix}$$

- Around y-axis

  X’ = zsinθ – xcosθ

  Z’ = zcosθ – xsinθ

$$\mathbf{R}_y = \begin{bmatrix} \cos\theta,0,\sin\theta \\ 0,1,0 \\ -\sin\theta,0,\cos\theta \end{bmatrix}$$

- Around z-axis

  X’ = xcosθ – ysinθ

  Y’ = xsinθ + ycosθ

$$\mathbf{R}_z = \begin{bmatrix} \cos\theta,-\sin\theta,0 \\ \sin\theta,\cos\theta,0 \\ 0,0,1 \end{bmatrix}$$

Affine transformations (cont.)

Rotation

$$\mathbf{R_x R_y R_z} = \begin{bmatrix} CyCz, -CySz, Sy \\ SxSyCz + CxSz, -SxSySz + CxCz, -SxCy \\ -CxSyCz + SxSz, CxSySz + SxCz, CxCy \end{bmatrix}$$

Where
Sx = Sinθ_x and so on

Affine transformations (cont.)

Scaling

- C_s – center of scaling
- Let C_s=0 and **y**=0
- Then matrix is
- If *a*=b=c then it is uniform scaling

$$\mathbf{S}_{\mathbf{abc}} = \begin{bmatrix} a,0,0,0 \\ 0,b,0,0 \\ 0,0,c,0 \\ 0,0,0,1 \end{bmatrix}$$

Affine transformations (cont.)

- Reflection
- Shear
  - Shear by x-axis

$$\mathbf{H}_{\mathbf{x}} = \begin{bmatrix} 1,0,0 \\ a,1,0 \\ b,0,1 \end{bmatrix}$$

Combination of transformations

- $V_{new} = V_{old} \times (M_1 \times M_2 \times \ldots)$

- Lot of transformations in games
- So it is necessary hardware support

\- graphics processors